ローレンツ不変な高エネルギー理論の検証可能性

徳田　順生　(京都大学　天体核研究室D3)

指導教官: 田中貴浩 氏 (京都大学　天体核研究室 教授)

Ref.) JT　JHEP05(2019)216 (arXiv: 1902:10039)

2019 12.26　理論懇シンポジウム@NAOJ

Plan

- 1. Introduction ＆ Motivation (8 pages.)

  ・高エネルギー理論(UV completion)の情報を、**低エネルギー現象**のみから引き抜けるか?

  →Maybe yes　Point : “**Positivity bounds**”

  ※高エネルギー理論・・・よりミクロな世界も整合的に記述できる理論

- 2. Our work (6 pages.)

  ・UV completion の ユニタリー性, 解析性(↔因果律), ローレンツ不変性

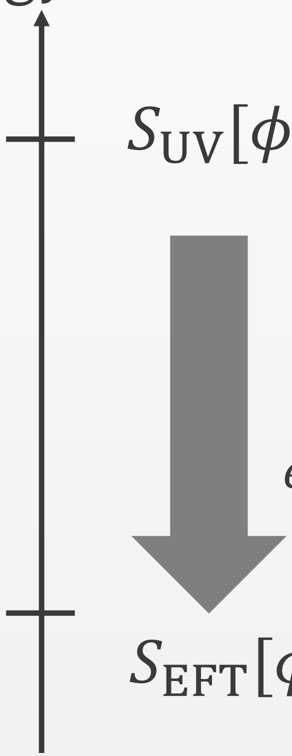
  という情報は低エネルギー有効理論に秘かに刻まれていそうである.

- 3. Summary (1 page.)

Quest for Fundamental theory…

- Phenomenology: Low-energy Effective Field Theory (EFT)

Energy scale



$S_{\mathrm{UV}}[\phi, \chi, \ldots]$ Fundamental theory (Unknown)

Heavy field ($E_{\mathrm{IR}} \ll M_\chi$) →integrated out

$$e^{iS_{\mathrm{EFT}}[\phi]} = \int \mathfrak{D}\mathrm{Heavy}\ e^{iS_{\mathrm{UV}}[\phi,\,\mathrm{Heavy}]}$$

Λ_{cut}

$S_{\mathrm{EFT}}[\phi]$ e.g.) : $\mathcal{L}_{\mathrm{EFT}} = -\frac{1}{2}(\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \cdots$

Constraints on EFT parameters → signals of new physics at $E \sim \Lambda_{\mathrm{cut}}$

Positivity bounds 1

- Assuming UV completion is

A. Adams *et al* ('06)

① Unitary $\hat{S}\hat{S}^{+} = 1$
② Lorentz invariant
③ Analytic (↔ Causal)
④ Local $[\hat{\phi}(x), \hat{\phi}(y)] = 0$ for spacelike $(x - y)$.

⇒ Low-energy EFT scattering amplitudes must satisfy an infinite number of inequalities.

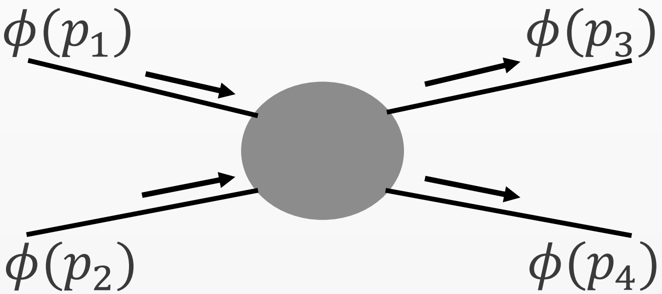
(※One has to introduce mass term as IR regulator.)

"positivity bounds"

Essential properties of UV completion ①-④ are **secretly encoded** in EFT !

Positivity bounds 2

- Low-energy 2 to 2 scat. amplitude $F(s,t)$ is constrained.



ϕ: scalar field with mass m

$= F(s,t)$ wl. Mandelstam variables

$s \equiv -(p_1+p_2)^2$: CM energy

$t \equiv -(p_1-p_3)^2$: momentum transfer

$u \equiv -(p_1-p_4)^2$

$s+t+u=4m^2$

$\cos\theta = 1 + \dfrac{2t}{s-4m^2}$

- Positivity bounds:

$$\partial_s^2 B(s,0)\Big|_{s=2m^2} > 0$$

$$\partial_s^4 B(s,0)\Big|_{s=2m^2} > 0$$

$$\partial_s^6 B(s,0)\Big|_{s=2m^2} > 0$$

$$\vdots$$

A. Adams *et al* ('06)

wl. $B(s,0) \equiv F(s,0) - (\text{light poles})$

e.g.) $\mathcal{L}_{\rm EFT} = -\frac{1}{2}(\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \cdots$

$c_1 > 0$

- トイモデルを考えてみると確かに$c_1 > 0$となる

Plan

- Assumptions on UV completion

① Unitary
② Lorentz invariant
③ Analytic (↔ Causal)
④ Local

Constraints on $F(s,t)$

- Behavior at high energy
- Analyticity as a complex function

§ 1. Review (5 pages)

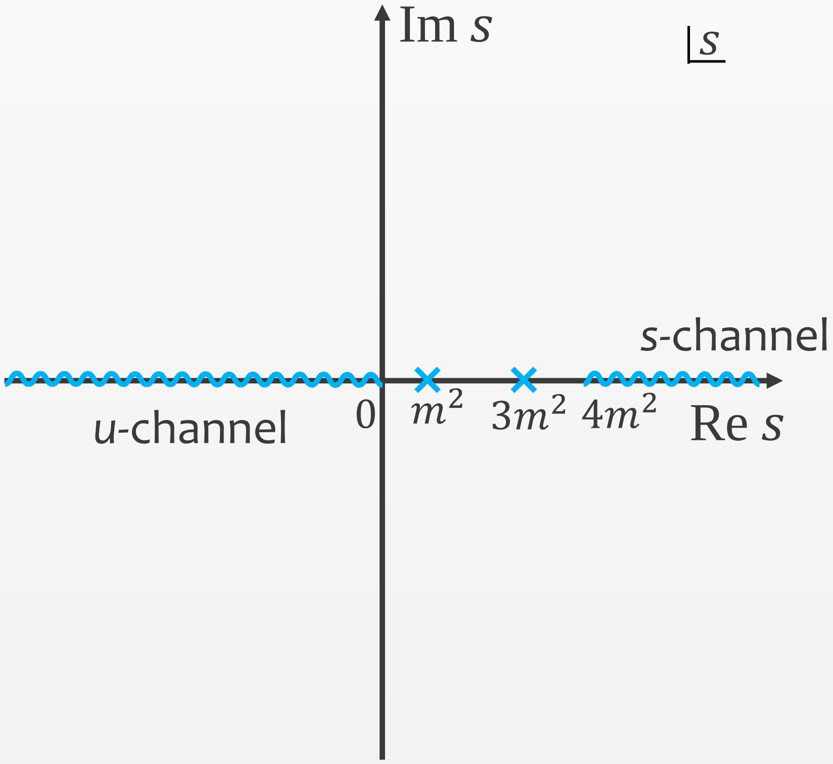
Constraints on EFT
"positivity bounds"

§ 2. Our study (7 pages)

What's happen if one removes locality assumption?
What is the definition of locality?

Analyticity and Causality (1/5)

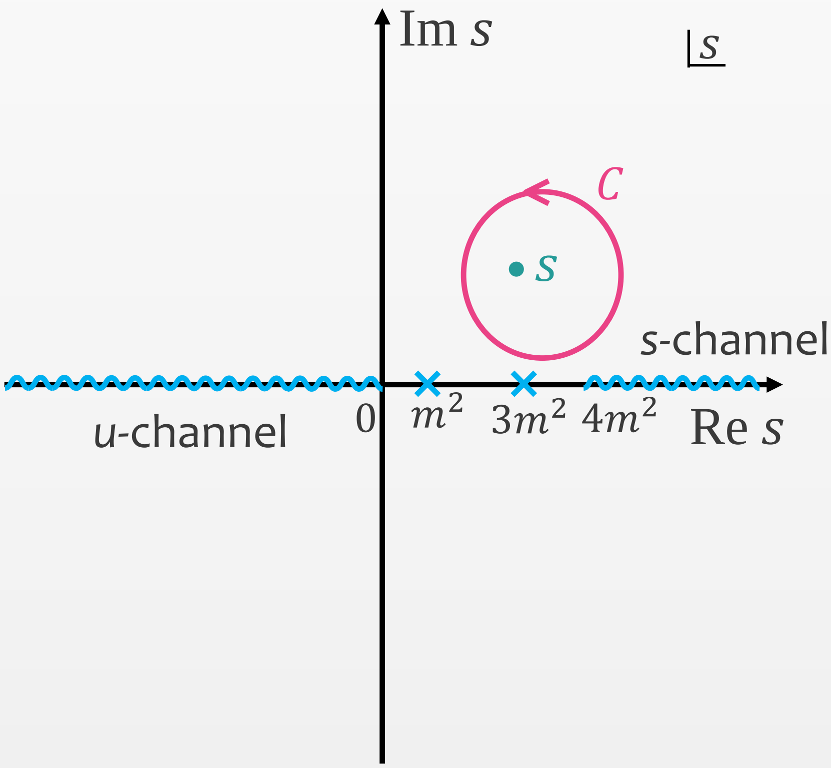
- Analytic structure of $F(s,0)$ in complex s-plane



Causality implies **the holomorphy of $F(s,0)$ in s**
in the complex s-plane modulo poles and cuts.

Derivation of Positivity bounds

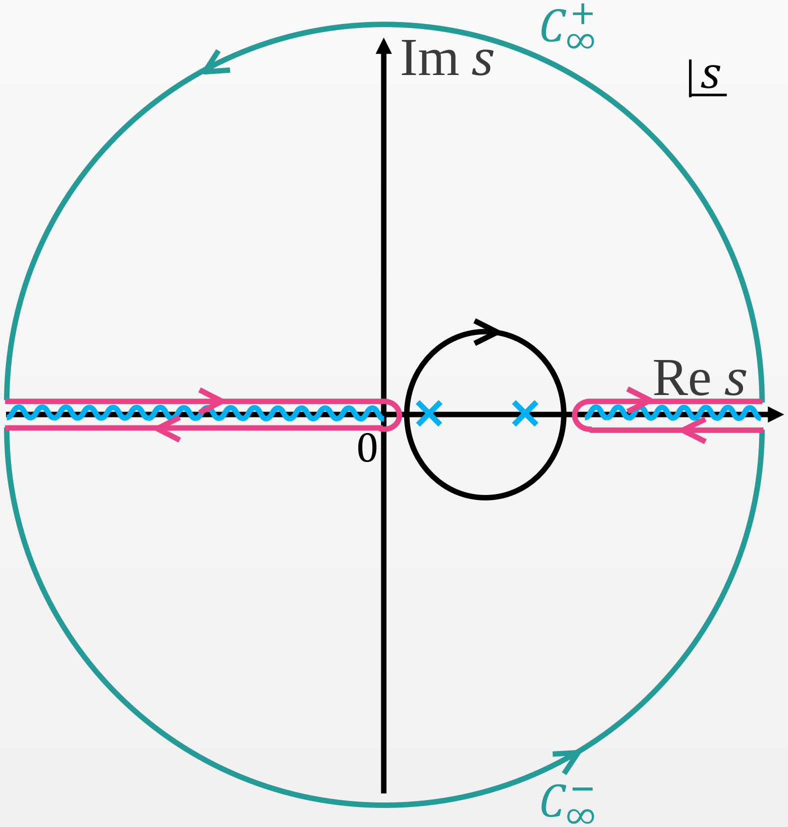
- Assuming the analytic structure of $F(s,0)$ in complex s-plane and derive the positivity bounds.



$$F(s,0) = \left(s - 2m^2\right)^{2N} \oint_C \frac{\mathrm{d}s'}{2\pi i} \frac{F(s',0)}{(s'-s)(s'-2m^2)^{2N}}$$

Derivation of Positivity bounds

- Analytic structure in complex s-plane

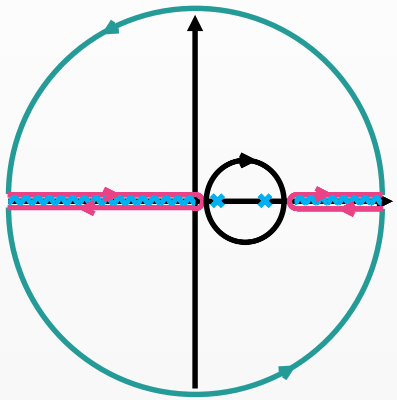


$$F(s,0) = \text{(light poles)}$$

$$+ \int_{\text{branch cuts}}$$

$$+ \int_{C_\infty^\pm}$$

Derivation of Positivity bounds



$$F(s,0) = (\text{light poles}) + \int_{\text{cuts}} \quad + \int_{C_\infty^\pm}$$

$$= \left[-\frac{\text{Res}_{s=m^2}F(s,0)}{m^2-s} + \frac{\text{Res}_{u=m^2}F(s,0)}{m^2-u}\right] + \sum_{k=0}^{2N-1} a_k s^k$$

$$+\frac{2(s-2m^2)^{2N}}{\pi}\int_{4m^2}^{\infty} \mathrm{d}\mu \left(\frac{\text{Im}\,F(\mu+i\epsilon,0)}{(\mu-2m^2)^{2N-1}[(\mu-2m^2)^2-(s-2m^2)^2]}\right)$$

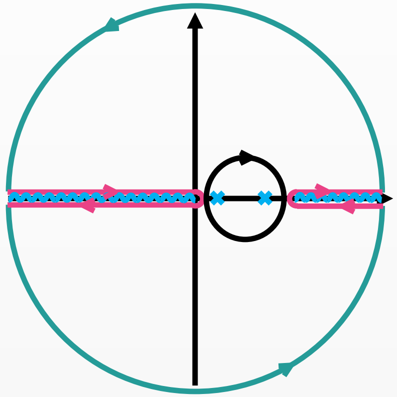
$$+(s-2m^2)^{2N}\int_{C_\infty^\pm}\frac{\mathrm{d}s'}{2\pi i}\frac{F(s',0)}{(s'-s)(s'-2m^2)^{2N}}$$

Locality & Unitarity $\rightarrow$ $\lim_{|s|\to\infty}\left|\frac{F(s,0)}{s^2}\right| = 0$ "Froissart (-like) bound"

M. Froissart (1961)

$\int_{C_\infty^\pm} = 0$ for $2N \geq 2$: 2-subtraction is sufficient.

Derivation of Positivity bounds



- Locality & Unitarity→2-subtraction

$$B(s,0) \equiv F(s,0) - (\text{light poles})$$

$$\partial_s^{2N} B(s,0)\Big|_{s=2m^2} = \frac{2(2N)!}{\pi}\int_{4m^2}^{\infty} \mathrm{d}\mu\, \frac{\mathrm{Im}\, F(\mu+i\epsilon,0)}{(\mu-2m^2)^{2N+1}} > 0$$

for $2N \geq 2$.

Optical theorem **(Unitarity)**

Locality & Unitarity : $\lim_{|s|\to\infty}\left|\frac{F(s,0)}{s^2}\right| = 0$

- Improved positivity bounds

$$\partial_s^{2N} B(s,0)\Big|_{s=2m^2} - \frac{2(2N)!}{\pi}\int_{4m^2}^{\Lambda_{\rm th}^2} \mathrm{d}\mu\, \frac{\mathrm{Im}\, F(\mu+i\epsilon,0)}{(\mu-2m^2)^{2N+1}} > 0$$

$\Lambda_{\rm th}$: cutoff scale of EFT

Positivity bounds without locality ??

- Positivity bounds are obtained:

① Unitarity
② Lorentz invariance
③ Analyticity
④ Locality

→

$$\partial_s^2 B(s,0)\Big|_{s=2m^2} > 0$$
$$\partial_s^4 B(s,0)\Big|_{s=2m^2} > 0$$
$$\partial_s^6 B(s,0)\Big|_{s=2m^2} > 0$$
$$\vdots$$

- 現状ではpositivity boundsが破れても、ユニタリーで解析的かつローレンツ不変なUV completion棄却/検証不可。

Is it impossible to derive these bounds only from ①-③?

① Unitarity
② Lorentz invariance
③ Analyticity

→ ?

Jaffe's classification of QFT

- Definition

A. M. Jaffe (1967)...

Growth rate of Lehmann-Källén spectral density $\rho(-k^2)$

$$\rho(-k^2) \sim (-k^2)^N \exp\left[\sigma(-k^2)^\alpha\right]$$

$0 \leq \alpha < \frac{1}{2}$: strictly localizable

$\alpha > \frac{1}{2}$: non-localizable

$\alpha = \frac{1}{2}$: quasi-local

e.g.) Little string theories : $\alpha = \frac{1}{2}$ A. Kapustin ('01)

Galileon theories : $\alpha > \frac{1}{2}$ A. J. Tolley *et al.* ('15)

- Position-space expression

$$W(x,y) := <\phi(x)\phi(y)> = \int \frac{\mathrm{d}^4 k}{(2\pi)^3}\theta(k^0)\,\rho(-k^2)\,e^{ik(x-y)} \text{ with } x \neq y$$

is **ill-defined** for $\alpha \geq \frac{1}{2}$.

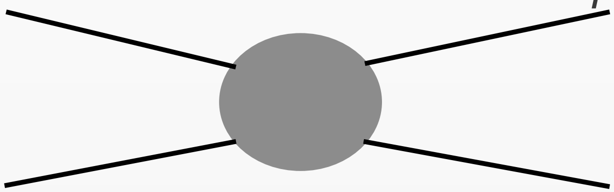
- Unitary S-matrix with standard properties (such as crossing symmetry, LSZ construction, etc) can be constructed.

O. Steinmann (1970)

Construct well-defined Feynman propagator (1/4)

E. Pfaffelhuber (1971)

- 2 to 2 scattering $\sim$ 4-point Feynman propagator



$\sim$ $< T\phi(x)\phi(y)\phi(z)\phi(w) >$

- Smeared Wightman function:

$$W_g(x,y) = \int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 k_2}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1 + k_2) \frac{\widetilde{W}_\phi(k_1, k_2)}{g(-k_1^2)} e^{i(k_1 x_1 + k_2 x_2)}.$$

$g(-k_1^2)$: an indicator function $\qquad g(-k_1^2) \sim \exp\left[\sigma(-k^2)^{\alpha}\right]$

- Then, we may define the regularized Feynman propagator:

$$G_F(x,y) = g(-i\partial_x, -i\partial_y)\left[\theta(x^0 - y^0) W_g(x,y) + \theta(y^0 - x^0) W_g(y,x)\right].$$

Construct well-defined Feynman propagator (2/4)

E. Pfaffelhuber (1971)

- Then, momentum-space Feynman propagator is

$$G_F(k_1, k_2) \equiv \int \mathrm{d}^4 x_1 \int \mathrm{d}^4 x_2 G_F(x_1, x_2) e^{-i(k_1 x_1 + k_2 x_2)}$$

$$= (2\pi)^4 \delta^{(4)}(k_1 + k_2) G_F(-k_1^2),$$

$$G_F(-k^2) = g(-k^2) \int \mathrm{d}\mu \frac{\rho(\mu)}{g(\mu)} \frac{-i}{k^2 + \mu - i\epsilon}.$$

an entire function which ensures the convergence of the integral.

- This spectral representation implies

$$\left|G_F(-k^2)\right| < \left|-k^2\right|^N e^{\sigma\left|-k^2\right|^\alpha} \quad \text{as } \left|-k^2\right| \to \infty.$$

Construct well-defined Feynman propagator (3/4)

E. Pfaffelhuber (1971)

- Indicator dependence=contact terms

$$G_F(-k^2) = g(-k^2) \int \mathrm{d}\mu \frac{\rho(\mu)}{g(\mu)} \frac{-i}{k^2 + \mu - i\epsilon}$$

$$= \int \mathrm{d}\mu\, \rho(\mu) \frac{-i}{k^2 + \mu - i\epsilon} \left(1 - \frac{g(\mu) - g(-k^2)}{g(\mu)} \right)$$

$$= \int \mathrm{d}\mu\, \rho(\mu) \frac{-i}{k^2 + \mu - i\epsilon} \left(1 - \frac{1}{g(\mu)} \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n g(-k^2)}{\partial^n(-k^2)} (\mu + k^2)^n \right)$$

- Indicator-dep terms: on-shell singularity is canceled.
- Scattering amplitudes are independent of the choice of an indicator function $g(\mu)$.

Construct well-defined Feynman propagator (4/4)

E. Pfaffelhuber (1971)

- Generalize to the 4-point case:

$$G_F(x_1,\cdots,x_4)=g\left(-i\partial_{x_1},\cdots,-i\partial_{x_4}\right)$$
$$\times\left[\sum_I \theta\left(x_{i_1}^0-x_{i_2}^0\right)\theta\left(x_{i_2}^0-x_{i_3}^0\right)\theta\left(x_{i_3}^0-x_{i_4}^0\right)W_g\left(x_{i_1},\cdots,x_{i_4}\right)\right].$$

- Similar to the 2-point case, momentum-space time-ordered correlation function will be bounded as

$$\left|\mathcal{G}_F\left(s,t,\{k_i^2\}_{i=1,\cdots,4}\right)\right|<|s|^N e^{\sigma|s|^\alpha}\quad \text{as } |s|\to\infty\ (t\text{: fixed}).$$

wl $G_F(k_1,\cdots,k_4)=(2\pi)^4\delta^{(4)}(k_1+\cdots+k_4)\mathcal{G}_F\left(s,t,\{k_i^2\}_{i=1,\cdots,4}\right)$

$$s\equiv-(k_1+k_2)^2\qquad t\equiv-(k_1-k_3)^2$$

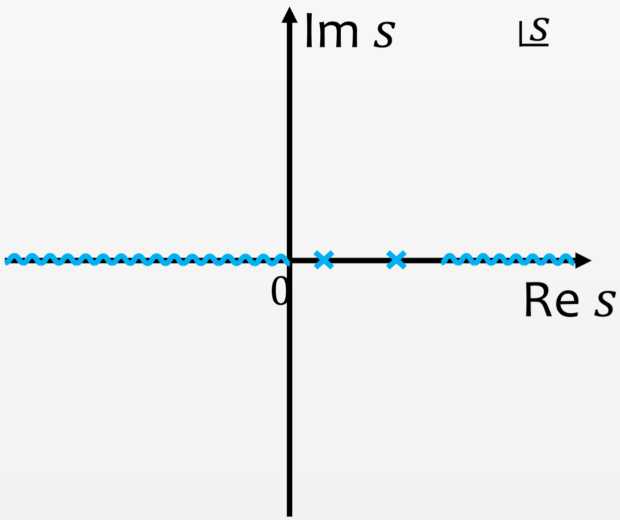
$\Longrightarrow$ $|F(s,t)|<|s|^N e^{\sigma|s|^\alpha}$ is expected (reduction formula).

$\alpha<\frac{1}{2}$ case $\rightarrow$ obtained   H. Epstein *et al.* (1969)

Assumptions

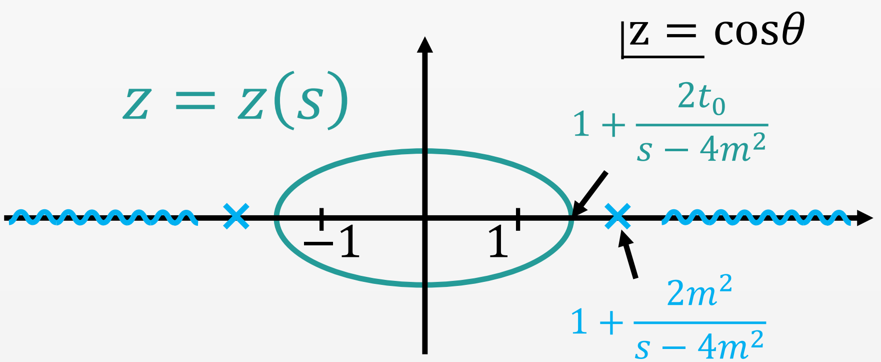
- We assume analytic structure and boundedness properties:

(A) Boundedness peopertiy of $F(s,0)$ in the complex s-plane.



$$\lim_{|s|\to\infty}\left|\frac{F(s,0)}{s^N e^{\sigma|s|^\alpha}}\right| = \text{finite}$$

(B) Boundedness property of $F(s,t)$ for fixed $t>0$.



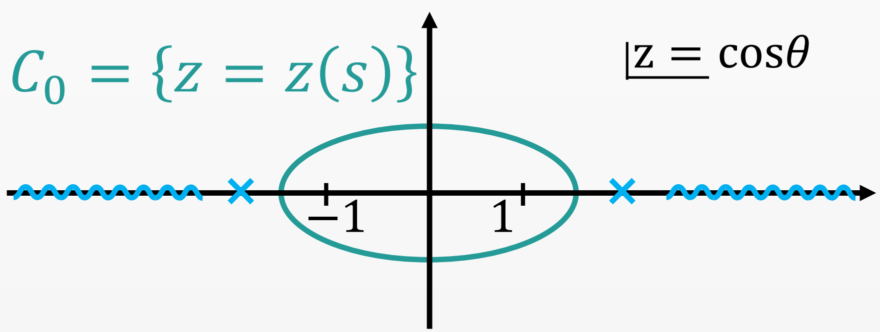
$$z = \cos\theta = 1+\frac{2t}{s-4m^2}$$

$$\lim_{s\to\infty}\left|\frac{\tilde{F}(s,z(s))}{s^N e^{\sigma s^\alpha}}\right| = \text{finite}$$

wl. $\tilde{F}(s,z) \equiv F(s,t)|_{2t=(s-4m^2)(z-1)}$

High-energy behavior (1/2)

- One can obtain bounds on partial wave amplitude $f_l(s)$



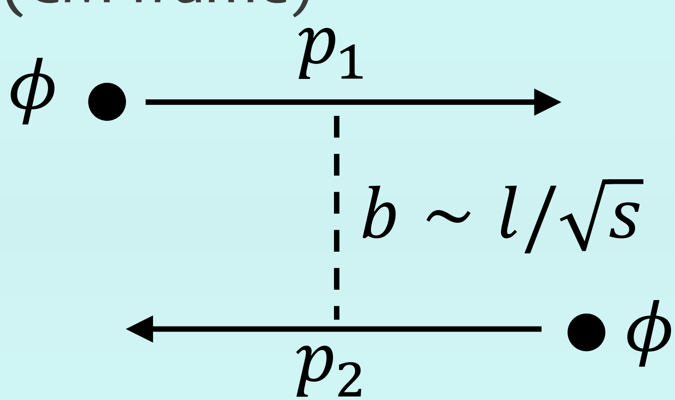
$$F(s,t) \sim \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l\left(1 + \frac{2t}{s-4m^2}\right)$$

$$f_l(s) = \frac{1}{4\pi i}\left(\frac{s-4m^2}{s}\right)^{\frac{1}{2}} \oint_{C_0} dz' \tilde{F}(s,z') Q_l(z')$$

Legendre functions of the second kind

$\Rightarrow$ $|f_l(s)| < (const.) \cdot l^{-\frac{1}{2}} \exp\left[-\frac{ml}{\sqrt{s}}\right] \exp[\sigma s^\alpha]$ at large l.

(CM frame)



Short-range force:

$$e^{-mb} \sim e^{-\frac{ml}{\sqrt{s}}}$$

Growth of the number of intermediate multi-particle states:

$$\rho(s) \sim \exp[\sigma s^\alpha]$$

High-energy behavior (2/2)

$$F(s,t) = 2\sqrt{\frac{s}{s-4m^2}}\sum_{l=0}^{\infty}(2l+1)f_l(s)P_l\left(1+\frac{2t}{s-4m^2}\right)$$

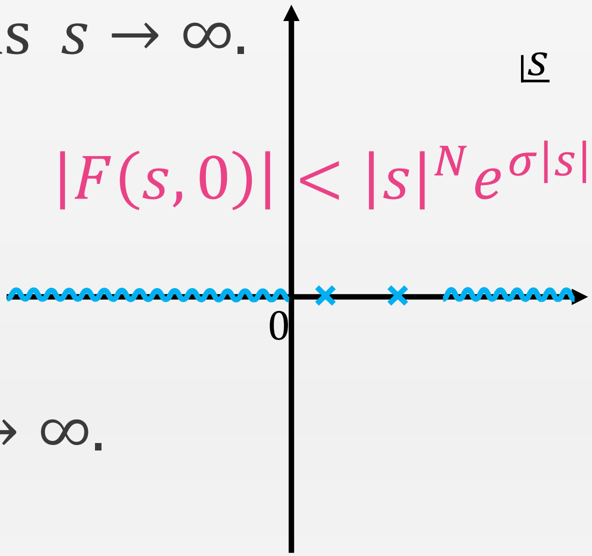
$$|f_l(s)| < (const.)\cdot l^{-\frac{1}{2}}\exp\left[-\frac{ml}{\sqrt{s}}\right]\exp[\sigma s^{\alpha}]$$ at large l.

$|f_l(s)| \leq 1$: unitarity bound (for $s > 4m^2$)

$$\Longrightarrow |F(s,0)| < \sum_{l=0}^{s^{\alpha+\frac{1}{2}}}(2l+1) \sim s^{1+2\alpha} \text{ as } s\to\infty.$$

Consistent with V. F. Fainberg *et al.* (1971)

$|F(s,0)| < |s|^N e^{\sigma|s|^{\alpha}}$



$$\Longrightarrow |F(s,0)| < |s|^{1+2\alpha}\ (\alpha < 1)\ as\ |s| \to \infty.$$

(thanks to the Phragmén-Lindelöf theorem)

Positivity bounds without locality

- Non-Locality & Unitarity→$2N > 1 + 2\alpha$-subtraction ($\alpha < 1$)

$$B(s,0) \equiv F(s,0) - (\text{light poles})$$

$$\partial_s^{2N} B(s,0)\Big|_{s=2m^2} = \frac{2(2N)!}{\pi}\int_{4m^2}^{\infty} d\mu \left(\frac{\operatorname{Im} F(\mu+i\epsilon,0)}{(\mu-2m^2)^{2N+1}}\right) > 0$$

for $2N > 1 + 2\alpha$. ⟵ $|F(s,0)| < |s|^{1+\alpha}$

- Existence of bounds for non-local case ($\frac{1}{2} \leq \alpha < 1$)

$$\partial_s^2 B(s,0)\Big|_{s=2m^2} > 0$$

$$\partial_s^4 B(s,0)\Big|_{s=2m^2} > 0$$

$$\partial_s^6 B(s,0)\Big|_{s=2m^2} > 0$$

$$\vdots$$

⟵ Must be satisfied even if theory is not strictly-local! (as long as $\alpha < 1$)

$0 \leq \alpha < \frac{1}{2}$: strictly localizable

$\alpha > \frac{1}{2}$: non-localizable

$\alpha = \frac{1}{2}$: quasi-local

Summary

- Locality will **not** be essential in obtaining positivity bounds.

① Unitarity
② Lorentz invariance
③ Analyticity
~~④ Locality~~

⇒

~~$\partial_s^2 B(s,0)\Big|_{s=2m^2} > 0$~~

$$\partial_s^4 B(s,0)\Big|_{s=2m^2} > 0$$

$$\partial_s^6 B(s,0)\Big|_{s=2m^2} > 0$$

$$\vdots$$

(as long as $\alpha < 1$)

- This may open a new possibility to falsify Lorentz invariant UV completion even if EFT is apparently Lorentz invariant.