Three-body calculations of the triple-alpha reaction rate at low temperatures

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1. INTRODUCTION

Triple-alpha reaction

\[ ^4\text{He} + ^4\text{He} + ^4\text{He} \rightarrow ^{12}\text{C} \]

- Resonant process \((T>10^8 \text{ K})\)
  \(^8\text{Be}, ^{12}\text{C}^*\)
  Resonance formula

- Non-resonant process \((T<10^8 \text{ K})\)

[A] Extension of the resonance formula with energy dependent widths.
  - NACRE [1]

[B] Quantum mechanical 3-body calculations
  - OKK: CDCC calculations (Ogata et al.[2])
  Significant effects at low temperature

\[ ^4\text{He} + ^4\text{He} \rightleftharpoons ^8\text{Be} \]
\[ ^8\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C}^*\left(0_2^+\right) \rightarrow ^{12}\text{C}\left(2_1^+\right) \rightarrow ^{12}\text{C}\left(0_1^-\right) \]

Astrophysical input: $3\alpha$ reaction rate $\langle\alpha\alpha\alpha\rangle$ [cm$^6$/s]

$$\dot{n}_{12} \equiv \frac{(n_4)^3}{6} \langle\alpha\alpha\alpha\rangle$$

$n_{12} (n_4)$: Number density of $^{12}$C ($^4$He)

Non-resonant

Resonant

$N_A^2 \langle\alpha\alpha\alpha\rangle$ (cm$^6$ s$^{-1}$ mol$^{-2}$)

$T_7 = T/(10^7$ K)
3-body calculations for $3\alpha$ reaction

- Ccontinuum Discretized Coupled Channel (CDCC)
  K. Ogata et al., PTP\textbf{122} (2009) 1055. [OKK]

- Hyperspherical Harmonics basis + R-matrix (HHR)

- This workshop:
  K. Yabana

  Ref: “Imaginary-time method for the radiative capture reaction rate”
  K. Yabana and Y.Funaki, PRC \textbf{85}, 055803 (2012)

- Faddeev
  S. Ishikawa, INPC2010, APFB2011, OMEG11
  (paper in preparation)
In the present talk:

- Calculation of $3\alpha$ reaction based on the Faddeev 3-body theory.
- Discussion about the difference from the OKK rate

CONTENTS

(1. Introduction)

2. Formalism

3. Calculations and Results

4. Discussion -- Comparison with CDCC results

5. Summary
2. FORMALISM
3α reaction \( \alpha + \alpha + \alpha \rightarrow ^{12}\text{C}(2^+) + \gamma \)

1. **Inverse reaction**: Photo induced 3α breakup of \(^{12}\text{C}(2^+)\)

\[
^{12}\text{C}(2^+) + \gamma \rightarrow \alpha + \alpha + \alpha
\]

2. Define a wave function for the breakup process

\[
|\Psi\rangle \equiv \frac{1}{E + i\epsilon - H_0 - V} H_\gamma |\Psi_b\rangle \rightarrow e^{iKR} \frac{\sigma^{(B)}(E_q, x, y)}{R^{5/2}} \\
R = \sqrt{x^2 + \frac{4}{3}y^2}
\]

3. Photodisintegration cross section

\[
\sigma_\gamma(E) \propto \int d\hat{x} d\hat{y} \int_{E_q > 0} dE_q \sqrt{E_q E_p} \left| f^{(B)}(E_q; \hat{x}, \hat{y}) \right|^2
\]

\[
E = E_q + E_p
\]
4. **Reaction rate**

\[
\langle \alpha\alpha\alpha \rangle = 240 (3)^{3/2} \pi \left( \frac{\hbar}{mc} \right)^3 c \int_0^\infty \frac{dEE'_\gamma}{(kT)^3} e^{-E/k_B T} \sigma_{12C(2^+_1)+\gamma \rightarrow 3\alpha} (E'_\gamma) \quad (E'_\gamma = E - E_{12C(2^+_1)})
\]

5. Apply the Faddeev formalism [1] to solve the equation for the 3-body disintegration process.

6. Apply the Sasakawa-Sawada method [2] to accommodate the long-range Coulomb interaction.

7. An approximation is made to treat a long-range contribution

Faddeev eq. (1961)

Multiple scattering with rearrangements

\[ \Phi^{(3)} (3,12) \]

\[ \Phi^{(1)} (1,23) \]

(Faddeev component)

\[ \Phi^{(2)} (2,31) \]

\[ \Psi (123) = \Phi^{(1)} (1,23) + \Phi^{(2)} (2,31) + \Phi^{(3)} (3,12) \]

Symmetric for 2<->3

Totally symmetric
An approximation

- A term \( \left( \frac{1}{x_3} - \frac{1}{y_1} \right) \) appeared in the integral kernel, which is expected to be short range because of a cancellation. But, the cancellation is not perfect for breakup channels.

\[ \rightarrow \text{treat this problem approximately by a (mandatory) cutoff procedure} \]

\[ \left( \frac{1}{x_3} - \frac{1}{y_1} \right) \times e^{-(x_3/R_{\text{cut}})^4} \]

\[ R_{\text{cut}} = 20 \text{ fm} - 35 \text{ fm} \]
3. CALCULATIONS AND RESULTS
3α model

• αα-potential
  Ali-Bodmer type (2-range Gaussian)

\[ V_{αα}(x) = \left( V_R^{(0)} \hat{P}_{L=0} + V_A^{(2)} \hat{P}_{L=2} \right) e^{-(x/a_R)^2} + V_A e^{-(x/a_A)^2} \]

<table>
<thead>
<tr>
<th></th>
<th>( a_R ) (fm)</th>
<th>( V_R^{(0)} ) (MeV)</th>
<th>( V_R^{(2)} ) (MeV)</th>
<th>( a_A ) (fm)</th>
<th>( V_A ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB(A')</td>
<td>1.53</td>
<td>125.0</td>
<td>20.0</td>
<td>2.85</td>
<td>-30.18</td>
</tr>
<tr>
<td>AB(D)</td>
<td>1.40</td>
<td>500.0</td>
<td>320.0</td>
<td>2.11</td>
<td>-130.0</td>
</tr>
</tbody>
</table>

• 3-body potential [1] to reproduce the binding energy and resonance energy

\[ V_{ααα} = \left( W_0 \hat{P}_{L=0} + W_2 \hat{P}_{L=2} \right) e^{-(\rho/3.9)^2} \quad \rho^2 = 3.97 \sum_{i=1}^{3} r_i^2 \]

Photodisintegration cross section
(AB-A’ & AB-D)

\[ \sigma_{12\text{C}(2^+)} + \gamma \rightarrow 3\alpha (E) \]

\[ E = E_\gamma + E_{12\text{C}(2^+)} \]

3\alpha energy in the cm system
$\alpha\alpha\alpha$ reaction rate (AB-A', AB-D)

$N_A^2 \langle <\alpha\alpha\alpha> \rangle_{\text{cm}^6 \text{s}^{-1} \text{mol}^{-2}}$

$\sim 10^{26}$ for OKK at $T_7=1$
4. DISCUSSION

Comparison with CDCC results
CDCC calculation of photo induced $3\alpha$ breakup of $^{12}\text{C}(2^+)$

$^{12}\text{C}(2^+) + \gamma \rightarrow \alpha + \alpha + \alpha$

Wave function for (photo-) disintegration process

$$|\Psi\rangle \equiv \frac{1}{E + i\epsilon - H_0 - V} H_\gamma |\Psi_b\rangle \quad \xrightarrow{R \to \infty} \quad \frac{e^{iKR}}{R^{5/2}} f^{(B)}(E_q, x, y)$$

$$(E - H_0 - V)|\Psi\rangle = H_\gamma |\Psi_b\rangle$$

Discretized $\alpha$-$\alpha$ functions $u_n(x)$:

$$[T_x + V_{\alpha\alpha}(x)]u_n(x) = E_{q_n} u_n(x)$$

$$\Psi(x, y) = \sum_n u_n(x) \phi_n(y)$$

$$\phi_n(y) \rightarrow [\text{Outgoing wave}] \times T_n$$

$$\sum_{n'} \left[ (E_p - T_y) \delta_{n,n'} - V_{n,n'}(y) \right] \phi_{n'}(y) = \langle u_n | H_\gamma | \Psi_b \rangle$$

$$\sigma_{\gamma}(E) \propto \sum_{n < n_0} \frac{|T_n|^2}{p_n}$$

# of base functions = 120 (~OKK)
Photodisintegration cross section (AB-A’)
(CDCC calculations by S.I.)

\[ \Gamma = 120 \text{ eV} \]

\[ \Gamma = 10 \text{ eV} \]

Single-channel cal.

\( \sigma_{\gamma}(E) \) (fm\(^2\))

\( E \) (MeV)
• At low temperatures (T<10^8 K):
\[<\alpha\alpha\alpha>_{\text{NACRE}} \sim <\alpha\alpha\alpha>_{\text{Faddeev}} \ll <\alpha\alpha\alpha>_{\text{CDCC}}\]

• Explanation of this enhancement by Ogata: Coulomb barrier between \(\alpha\alpha\)-pair and \(\alpha\): non-resonant pair vs. resonant pair
Reason for the enhancement (Ogata)

- Coulomb potential between $\alpha\alpha$-pair and $\alpha$-particle

\[ V_{(\alpha\alpha)-\alpha}(R) \text{ [MeV]} \]

![Graph showing the potential as a function of distance R, with non-resonant and resonant states depicted.](image-url)
Model space of CDCC calculation

• Only one set of Jacobi coordinate is used:
  Neglects of rearrangement channels as well as symmetrization of the wave functions
Rearrangement effect

Non-resonant state

Resonant state
3α decay mechanism of the Hoyle state

- The enhancement of $\sigma_\gamma(E)$ by the CDCC calculation at low energies is due to the reduction of Coulomb barrier between $\alpha$ and non-resonant $\alpha\alpha$-pair.

- This reduction may cause an enhancement of non-resonant (direct) process of 3α-decay of the Hoyle state.
Sequential decay:

\[ ^{12}\text{C}(2_1^+) + \gamma \rightarrow ^{12}\text{C}^*(0_2^+) \rightarrow ^8\text{Be} + ^4\text{He} \rightarrow ^4\text{He} + ^4\text{He} \]

Direct decay:

\[ ^{12}\text{C}(2_1^+) + \gamma \rightarrow ^{12}\text{C}^*(0_2^+) \rightarrow ^4\text{He} + ^4\text{He} + ^4\text{He} \]
3α decay of the Hoyle state

- Direct decay or Sequential two-step process

  $^{40}$Ca + $^{12}$C at 25MeV/nucleon
  Direct-decay contribution: 7.5 ± 4.0 %

  $^{11}$B($^{3}$He,d)
  “no evidence for direct-decay branches”

  $^{10}$C + $^{12}$C “An upper limit of 0.45%”
Decomposition of the cross section

\[ \sigma_{\gamma} (E) \propto \iint d\hat{x} d\hat{y} \int_{E_q > 0} dE_q \sqrt{E_q E_p} \left| f \left( E_q ; \hat{x}, \hat{y} \right) \right|^2 \]

\[ \sigma_{\gamma}^{R} (E) \propto \iint d\hat{x} d\hat{y} \int dE_q \sqrt{E_q E_p} \left| f \left( E_q ; \hat{x}, \hat{y} \right) \right|^2 \]

\[ \sigma_{\gamma}^{NR} (E) = \sigma_{\gamma} (E) - \sigma_{\gamma}^{R} (E) \]
Faddeev vs. CDCC (SI)

- $3\alpha$-decay of the Hoyle state

Sequential decay vs. Direct decay

--Faddeev: Sequential decay-dominant

--CDCC: large contribution from Direct decay

67% at E=380keV
Non-resonant contribution (CDCC)
Non-resonant contribution (Faddeev)
5. SUMMARY

- Quantum mechanical 3-body calculations of 3\(\alpha\)-reaction as photodisintegration of \(^{12}\text{C}(2^+)\)
  - Faddeev method, CDCC method
- Faddeev calculation: similar to the NACRE 3\(\alpha\) rate
- CDCC calculations: Increase of the cross section at low energies (similar to Ogata’s CDCC results)
- 3\(\alpha\)-decay of Hoyle resonance
  - Faddeev: Sequential decay (via \(^8\text{Be}\)) dominant
  - CDCC: A large contribution from the Direct decay
  \(\rightarrow\) This may be tested by experiments.

- Future problem:
  - Higher energies (theoretical calculations of \(^{12}\text{C}\)-resonance other than the Hoyle state)
    - \(^9\text{Be}(\alpha-\alpha-n), ~^{6}\text{He}(\alpha-n-n), n-n-n (3-n potential)\)
  - 4\(\alpha\) problem, \(^{12}\text{C}(\alpha,\gamma)^{16}\text{O}\)