ÖAW AUSTRIAN ACADEMY OF SCIENCES
IWF SPACE RESEARCH INSTITUTE

Monte Carlo Generation of Kappa Distributions in Kinetic Plasma Simulations

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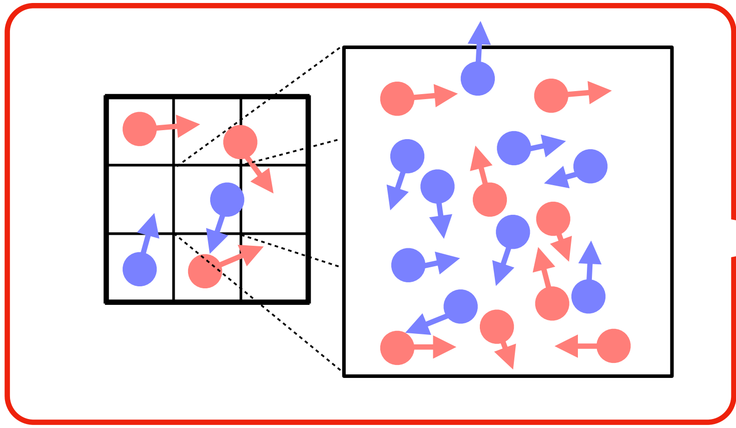
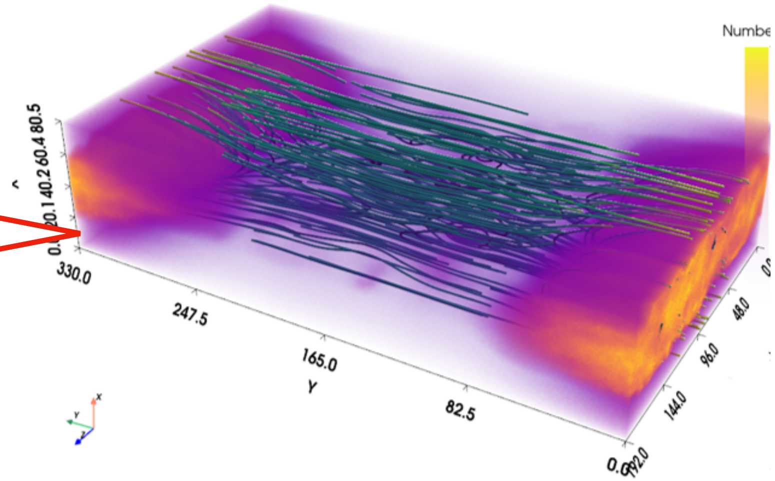
- 1. Kappa generators
- 2. Relativistic Kappa distribution
- 3. KLC (Kappa loss-cone) distributions
- 4. Gamma distribution

Particle-in-cell (PIC) simulation





1. Particle loader

2. Particle integrator

$$\boldsymbol{x}_n \to \boldsymbol{x}_{n+1}$$
$$\boldsymbol{v}_{n+\frac{1}{2}} \to \boldsymbol{v}_{n+\frac{3}{2}}$$

3. Charge deposition

$$x_n \to \overline{\rho}_{c,n}$$
$$\boldsymbol{v}_{n+\frac{1}{2}} \to \overline{\boldsymbol{j}}_{n+\frac{1}{2}}$$

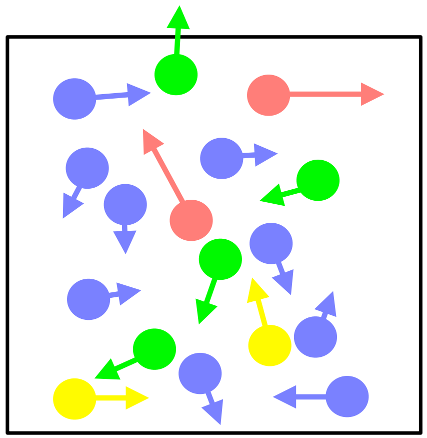
4. Field solver

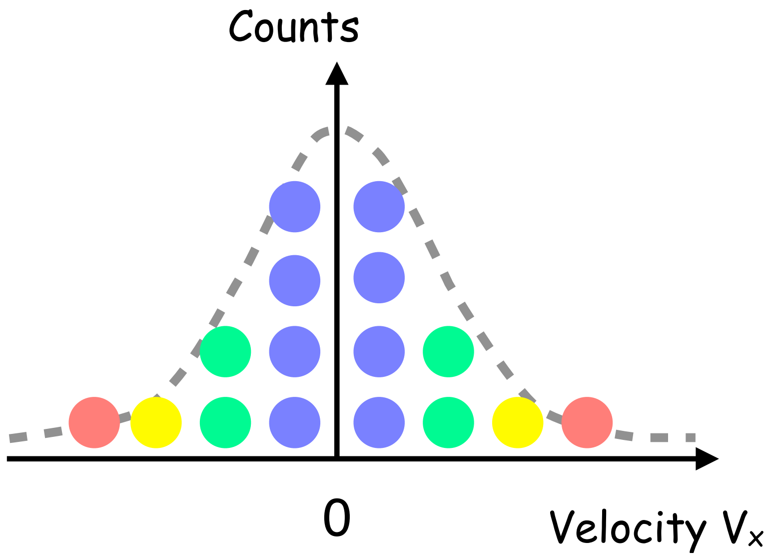
$$\boldsymbol{E}_n \to \boldsymbol{E}_{n+1}$$
$$\boldsymbol{B}_n \to \boldsymbol{B}_{n+1}$$

5. Field interpolation

$$\boldsymbol{E}'_n$$
$$\boldsymbol{B}'_n$$

Velocity distribution in PIC simulations

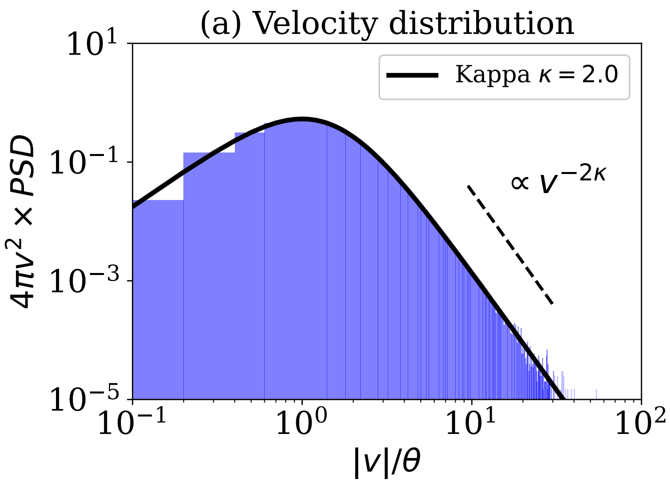




- Maxwell = Normal distribution
  - randn() routine, Box-Muller (1958) method

How to generate Kappa-type (Kappa, kappa-loss-cone, relativistic...) distributions in PIC simulation?

Kappa distribution



- Popular form

$$f(\boldsymbol{v})d^3v = \frac{N_\kappa}{(\pi\kappa\theta^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1 + \frac{\boldsymbol{v}^2}{\kappa\theta^2}\right)^{-(\kappa+1)} d^3v$$

  - Maxwellian-like core + power-law tail

- Random number generation methods
  - Standard method (t-generator)
  - Pareto method

    } Rejection sampling (not GPU-friendly)
  - Bailey method
  - Galton board method

    } 1D/2D only
  - Approximate Kappa generator for GPUs

Standard method: t-generator

Kappa

$$f(\boldsymbol{v})d^3v = \frac{N}{(\pi\kappa\theta^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}d^3v$$

$$\longleftrightarrow \quad \frac{N}{\nu^{p/2}\pi^{p/2}\sigma^p}\frac{\Gamma[(\nu+p)/2]}{\Gamma(\nu/2)}\left(1+\frac{1}{\nu}\frac{\boldsymbol{t}^2}{\sigma^2}\right)^{-\frac{(\nu+p)}{2}}$$

multivariate t-distribution

- Numerical procedure

$$n_1, n_2, n_3 \sim \mathcal{N}(0,1)$$

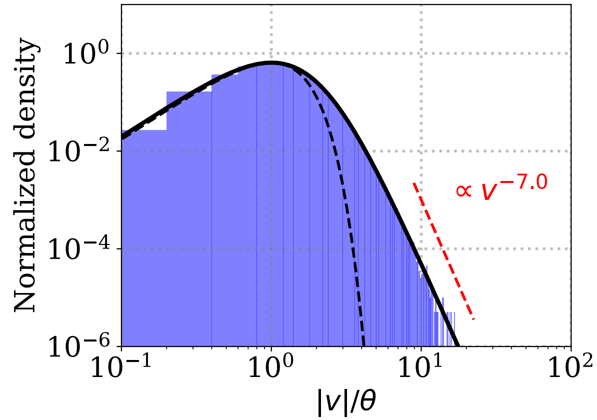
Normal random number

$$x \sim \mathrm{Ga}(\kappa - 1/2, 2)$$

Gamma random number

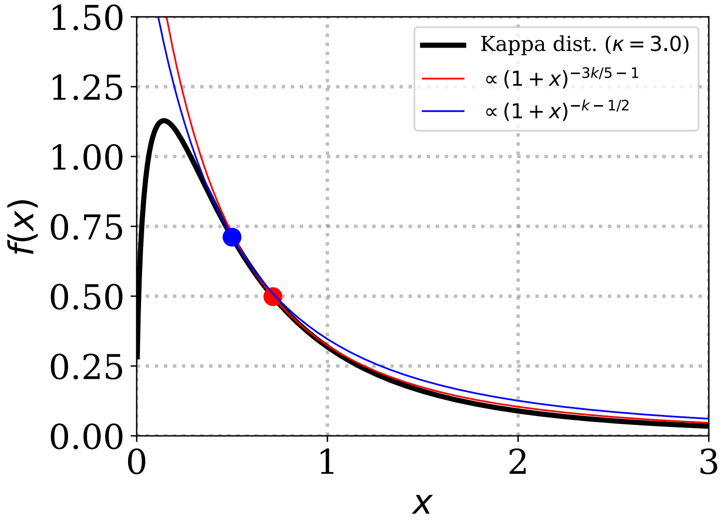
$$v_x \leftarrow \sqrt{\kappa\theta^2}\,\frac{n_1}{x}$$

Division by gamma



See also Abdul & Mace 2015, SZ & Nakano 2022

Pareto method



$\mathcal{D} \leftarrow \sqrt{(\kappa-1)^{\kappa-1}/\kappa^{\kappa}}$

repeat

  generate $U_1, U_2 \sim U(0,1)$

  $W \leftarrow \sqrt{(1-U_1)^{-2/\kappa}-1}$

until $W(1-U_1) > \mathcal{D}U_2$

generate $U_3, U_4 \sim U(0,1)$

$V \leftarrow \sqrt{\kappa\theta^2}W$

$v_x \leftarrow V(2U_3-1)$

$v_y \leftarrow 2V\sqrt{U_3(1-U_3)}\cos(2\pi U_4)$

$v_z \leftarrow 2V\sqrt{U_3(1-U_3)}\sin(2\pi U_4)$

return v_x, v_y, v_z

- Kappa, rescaled by energy

$$f(x)\,dx = \frac{1}{\mathrm{B}\left(\frac{3}{2}, \kappa-\frac{1}{2}\right)} x^{1/2}(1+x)^{-\kappa-1}\,dx$$

- Type II Pareto distribution --> Rejection sampling (75-80%)

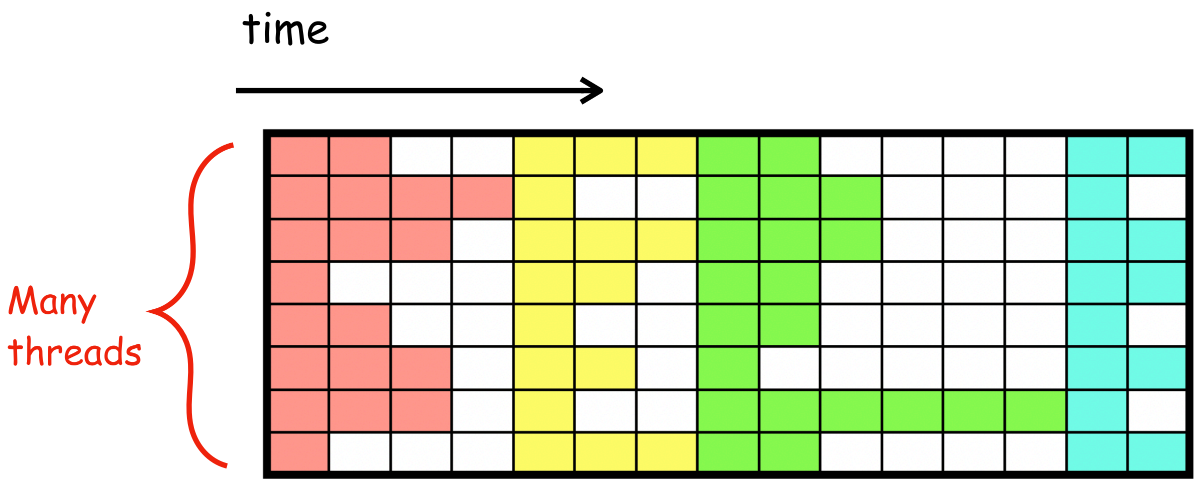
$$g(x) \equiv n(1+x)^{-(n+1)}$$

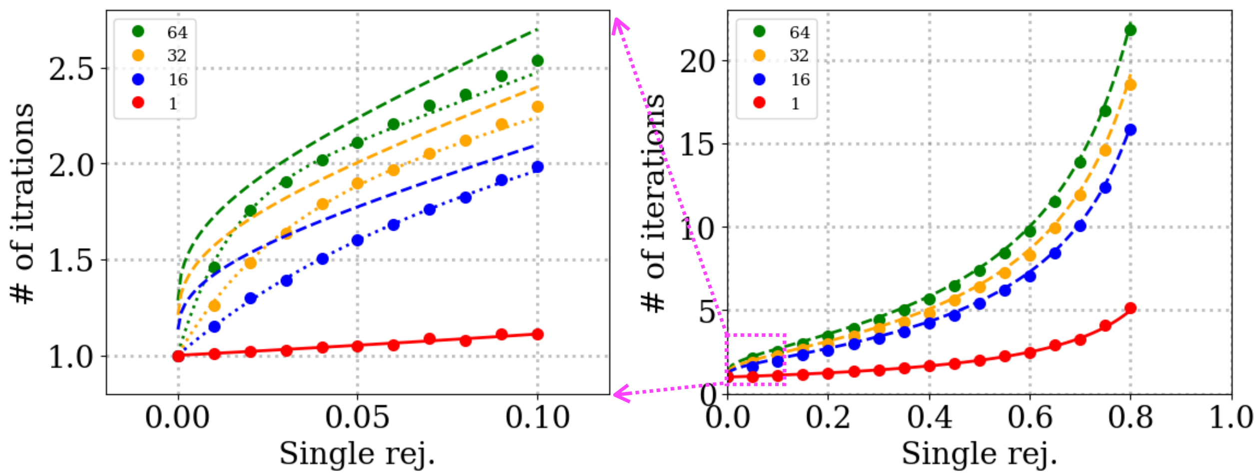
- No need for a Gamma random number

SZ 2025 RNAAS

A problem of a GPU execution model

- A single instruction controls 32 or 64 threads simultaneously (SIMT model)
- Significant slowdown for independent loops





Ridley & Forget 2021
Chakraborty & Gupta 2015

Approximate Kappa generator (1/4)

- Kappa distribution

$$f(\boldsymbol{v})d^3v = \frac{N_\kappa}{(\pi\kappa\theta^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1 + \frac{\boldsymbol{v}^2}{\kappa\theta^2}\right)^{-(\kappa+1)} d^3v$$

- Cumulative distribution function (CDF)

$$F(x) = I_{\frac{x}{x+\kappa}}\left(\frac{3}{2}, \kappa - \frac{1}{2}\right)$$

$I_x()$: Regularized incomplete beta function

$$\approx G(x) \equiv \left\{1 - \exp_{q*}\left(-\frac{ax+bx^2}{1+cx}\right)\right\}^{3/2}$$

- q-exponential function

$$\exp_q(x) \equiv \Big(1 + (1-q)x\Big)^{\frac{1}{1-q}} \quad \longleftrightarrow \quad \ln_q(x) \equiv \frac{x^{1-q}-1}{1-q}$$

Approximate Kappa generator (2/4)

- Kappa distribution

$$f(\boldsymbol{v})d^3v = \frac{N_\kappa}{(\pi\kappa\theta^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{\boldsymbol{v}^2}{\kappa\theta^2}\right)^{-(\kappa+1)}d^3v$$

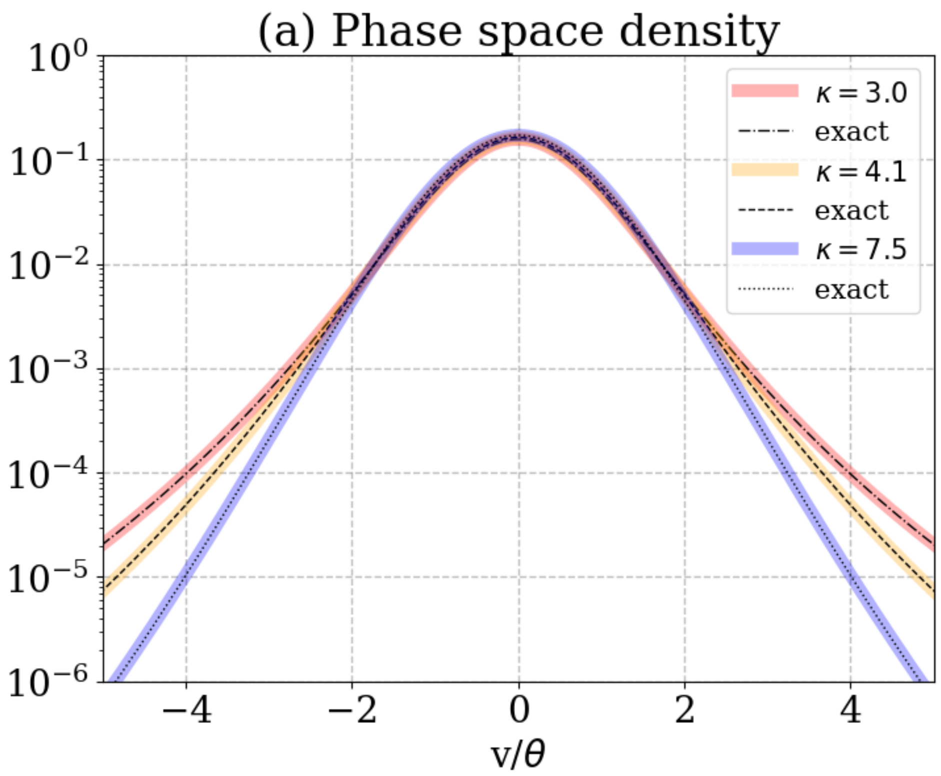
- Approx. Kappa distribution

$$g(\boldsymbol{v})d^3v = \frac{3N_\kappa(a\theta^4+2bv^2\theta^2+bcv^4)}{4\pi v\theta^2\left(\theta^2+cv^2\right)^2}$$
$$\times\sqrt{1-\left(1+\frac{(a\theta^2+bv^2)v^2}{\kappa^*(\theta^2+cv^2)\theta^2}\right)^{-\kappa^*}}$$
$$\times\left(1+\frac{(a\theta^2+bv^2)v^2}{\kappa^*(\theta^2+cv^2)\theta^2}\right)^{-(\kappa^*+1)}d^3v$$

Inverse-transform procedure
(No iteration loop)

$$\kappa^* \leftarrow \kappa - 1/2$$
$$a \leftarrow \frac{1}{\kappa}\left(\frac{2}{3B(3/2,\kappa^*)}\right)^{2/3}$$
$$c \leftarrow \frac{0.123\kappa^2-1.12\,\kappa+2.56}{\kappa^2-7.89\,\kappa+15.6}$$
$$b \leftarrow \left(\kappa^*\tfrac{3}{2}B(3/2,\kappa^*)\right)^{1/\kappa^*}\frac{\kappa^*}{\kappa}\,c$$
$$\text{generate } U_1, U_2, U_3 \sim \text{Uniform}(0,1)$$
$$L \leftarrow -\kappa^*\left\{\left(1-U_1^{2/3}\right)^{-1/\kappa^*}-1\right\}$$
$$V \leftarrow \theta\sqrt{\frac{-2L}{(a+cL)+\sqrt{(a+cL)^2-4bL}}}$$
$$v_x \leftarrow V(2U_2-1)$$
$$v_y \leftarrow 2V\sqrt{U_2(1-U_2)}\cos(2\pi U_3)$$
$$v_z \leftarrow 2V\sqrt{U_2(1-U_2)}\sin(2\pi U_3)$$

Approx. Kappa (3/4): Comparison with exact Kappa



- δ(PSD) < PIC noise

  $$N_p \lesssim 10^7$$

- Total number

  same

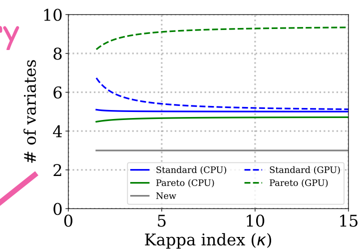
- Total energy

  $$\frac{\Delta\mathcal{E}}{\mathcal{E}} \lesssim 10^{-2.5} \quad (\kappa \approx 4.1)$$

  $$\frac{\Delta\mathcal{E}}{\mathcal{E}} \lesssim 10^{-3.5} \quad (\kappa \neq 4.1)$$

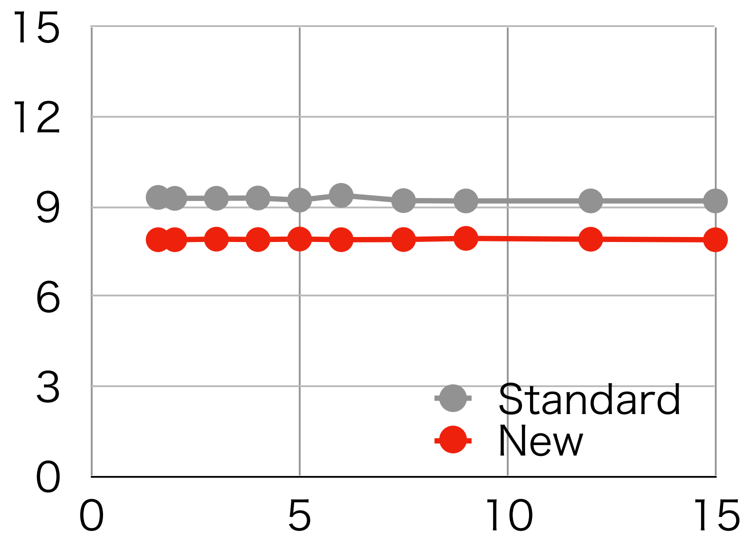
Approx. Kappa (4/4): benchmark

Theory



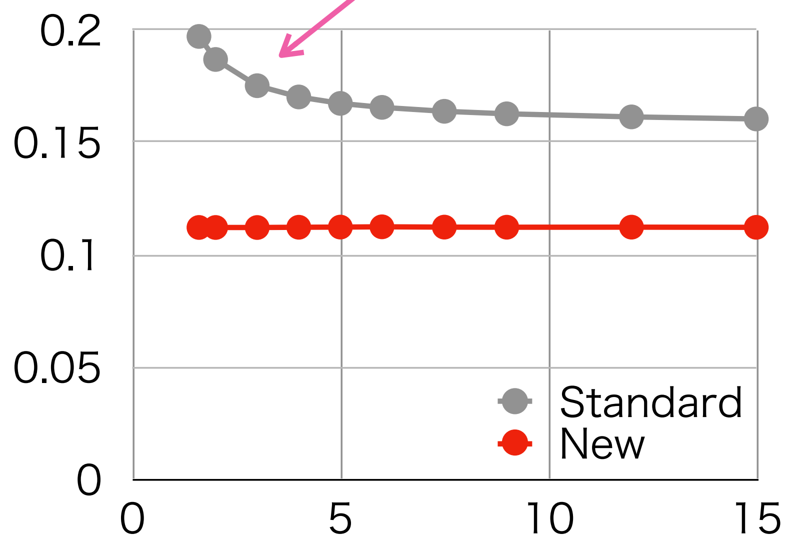
- CPU

Fast



Kappa index

- Nvidia GPU



Kappa index

- Although the Standard method suffers from the SIMT problem, New method runs constantly fast
- SIMT-aware algorithms are useful in plasma simulations

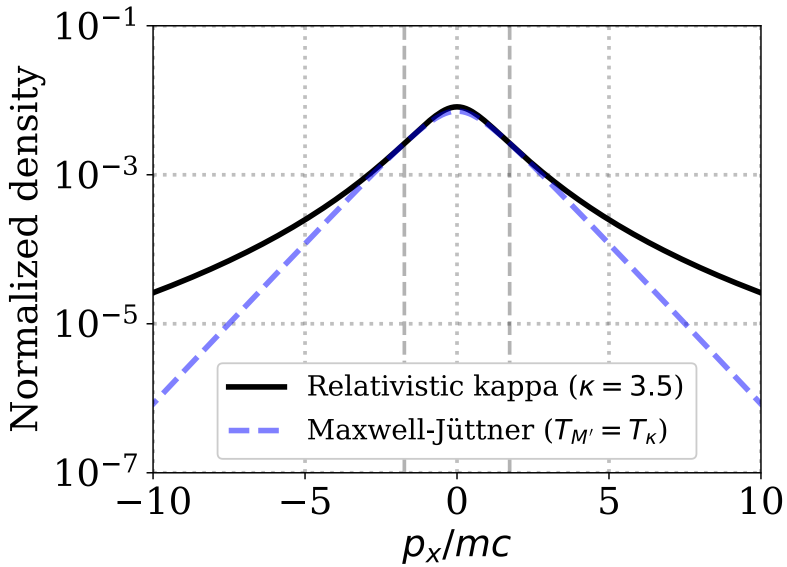
Relativistic Kappa (1/4)

$$f_{\rm RK}(\boldsymbol{p})d^3p \equiv \boxed{A}\left(1+\frac{(\gamma-1)mc^2}{\kappa T_\kappa}\right)^{-(\kappa+1)} d^3p$$

$$\gamma = [1-(v/c)^2]^{-1/2}$$
$$\boldsymbol{p} = m\gamma\boldsymbol{v}$$

Normalization constant

$$A(\kappa, T_\kappa) = \frac{N_\kappa \Gamma\left(\kappa+\frac{1}{2}\right)}{(2\pi m\kappa T_\kappa)^{3/2}(\kappa+1)\ \Gamma(\kappa-2)\ {}_2F_1\left(-\frac{3}{2},\frac{5}{2};\kappa+\frac{1}{2};1-\frac{\kappa T_\kappa}{2mc^2}\right)}$$



- Not so popular yet
  - PIC discussion started only recently (e.g., Zhdankin 2022)
  - See Han-Thanh+ 2022 for mathematical detail

Relativistic Kappa (2/4): mathematics

- Relativistic Kappa distribution

$$\boxed{f_{\mathrm{RK}}(p)dp = A(\kappa,t)\left(1+\frac{\gamma-1}{\kappa t}\right)^{-(\kappa+1)} 4\pi p^2\, dp}$$

$$x \equiv \frac{\mathcal{E}_{\mathrm{kin}}}{mc^2} = \gamma - 1$$

$$A(\kappa,T_\kappa) = \frac{N_\kappa \Gamma\left(\kappa+\frac{1}{2}\right)}{(2\pi m\kappa T_\kappa)^{3/2}(\kappa+1)\ \Gamma(\kappa-2)\ {}_2F_1\left(-\frac{3}{2},\frac{5}{2};\kappa+\frac{1}{2};1-\frac{\kappa T_\kappa}{2mc^2}\right)}$$

$$S(\kappa,t) \equiv \frac{\sqrt{2\pi}}{2}\Gamma\left(\kappa-\frac{1}{2}\right) + a\sqrt{\kappa t}\ \Gamma(\kappa-1) + b\,\frac{3\sqrt{2\pi}}{4}(\kappa t)\left(\kappa-\frac{3}{2}\right) + 2(\kappa t)^{3/2}\ \Gamma(\kappa-2)$$

- Our expression

Normalization factors

$$\boxed{f_{\mathrm{RK}}(x)dx = \frac{4\pi A(\kappa,t)S(\kappa,t)}{\Gamma(\kappa+1)}(\kappa t)^{3/2}\left(\sum_{i=3}^{6}\pi_i(\kappa,t)\ \mathrm{B}'\left(x;\frac{i}{2},\kappa+1-\frac{i}{2},1,\kappa t\right)\right)R(x)\ dx}$$

Probability

$$\sum_{i=3}^{6}\pi_i(\kappa,t) = 1$$

Generalized Beta-prime distribution

$$\mathrm{B}'(x;\alpha,\beta,p,q) = \frac{p}{qB(\alpha,\beta)}\left(\frac{x}{q}\right)^{\alpha p-1}\left(1+\left(\frac{x}{q}\right)^p\right)^{-(\alpha+\beta)}$$

Rejection function

$$R(x;a,b) \equiv \frac{(1+x)\sqrt{x+2}}{\sqrt{2}+ax^{1/2}+b\sqrt{2}x+x^{3/2}}$$

Relativistic Kappa (3/4): Recipe

Main procedure

$a \leftarrow 0.56, \quad b \leftarrow 0.35, \quad R_0 \leftarrow 0.95$

compute π_3, π_4, π_5 for given κ, t using Eqs. (40)–(42)

repeat

generate $X_1, X_2 \sim U(0,1)$

Probabilistic switch

if $X_1 < \pi_3$ **then** $i \leftarrow 3$

elseif $X_1 < \pi_3 + \pi_4$ **then** $i \leftarrow 4$

elseif $X_1 < \pi_3 + \pi_4 + \pi_5$ **then** $i \leftarrow 5$

else $i \leftarrow 6$

endif

generate $X_3 \sim \mathrm{Ga}(i/2, 1), X_4 \sim \mathrm{Ga}(\kappa + 1 - i/2, 1)$

$x \leftarrow \kappa t \times \dfrac{X_3}{X_4}$ Beta-prime distribution

until $X_2 < R_0$ **or** $X_2 < R(x; a, b)$ Rejection

generate $X_5, X_6 \sim U(0,1)$

$p \leftarrow \sqrt{x\,(x+2)}$

$p_x \leftarrow p\ (2X_5 - 1)$

$p_y \leftarrow 2p\sqrt{X_5(1-X_5)}\cos(2\pi X_6)$

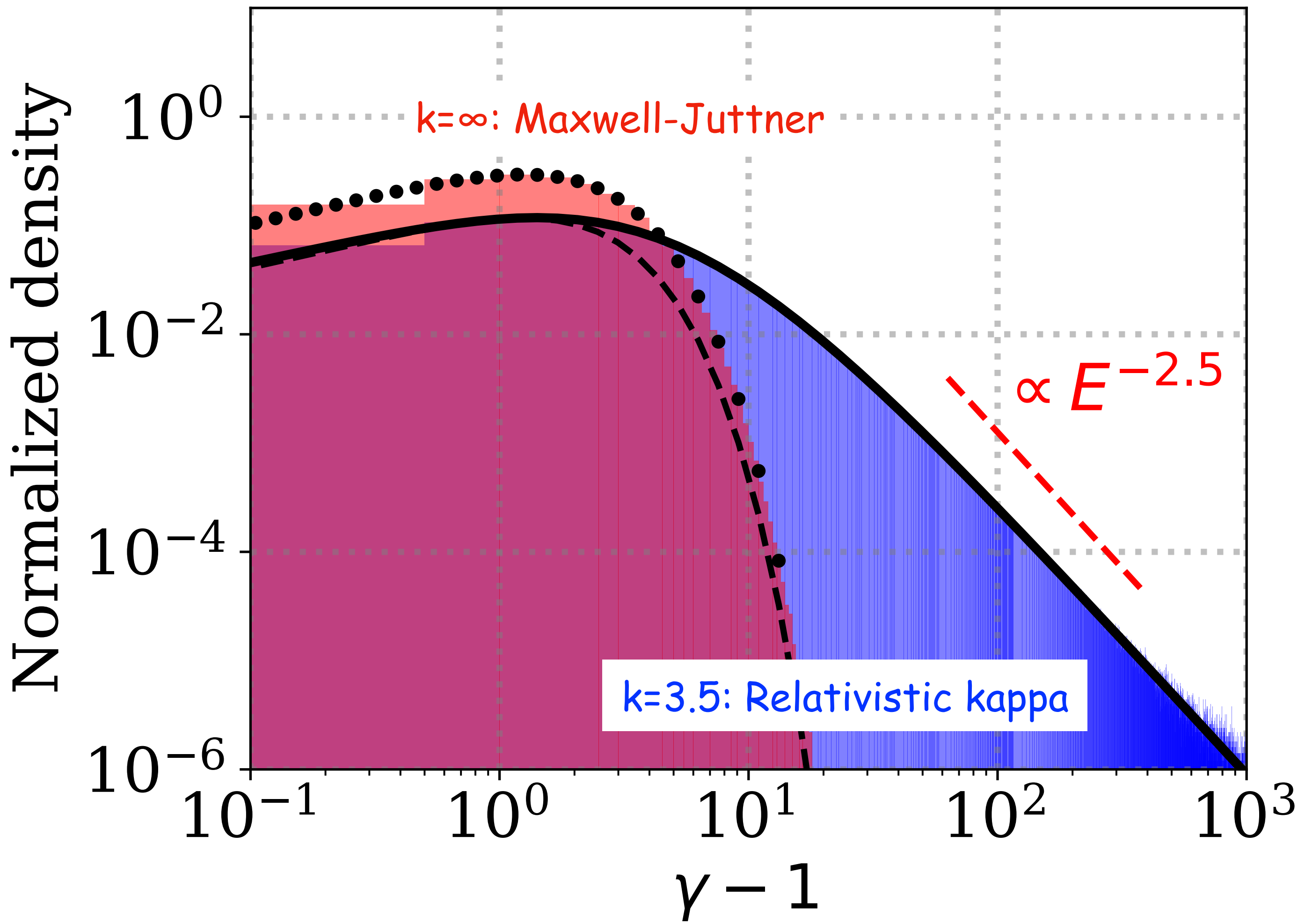
$p_z \leftarrow 2p\sqrt{X_5(1-X_5)}\sin(2\pi X_6)$

- No need for the normalization factor
- Beta-prime distribution can be generated from 2 gamma distributions
- Gamma variates can be easily generated

$$X_{\mathrm{B}'(\alpha,\beta)} = \frac{X_{\Gamma(\alpha,1)}}{X_{\Gamma(\beta,1)}}$$

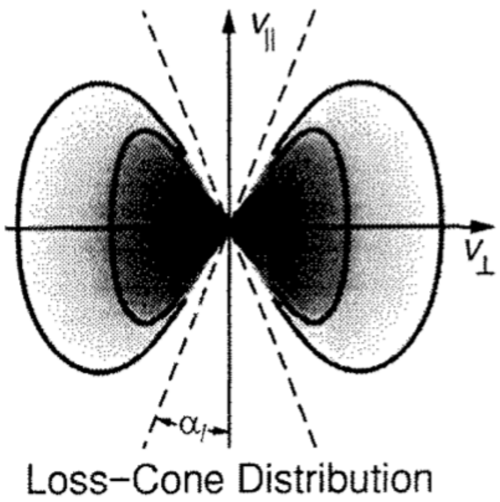
- The rejection process typically works for 0~4% cases

Relativistic Kappa (4/4): Numerical test



SZ & Nakano 2022

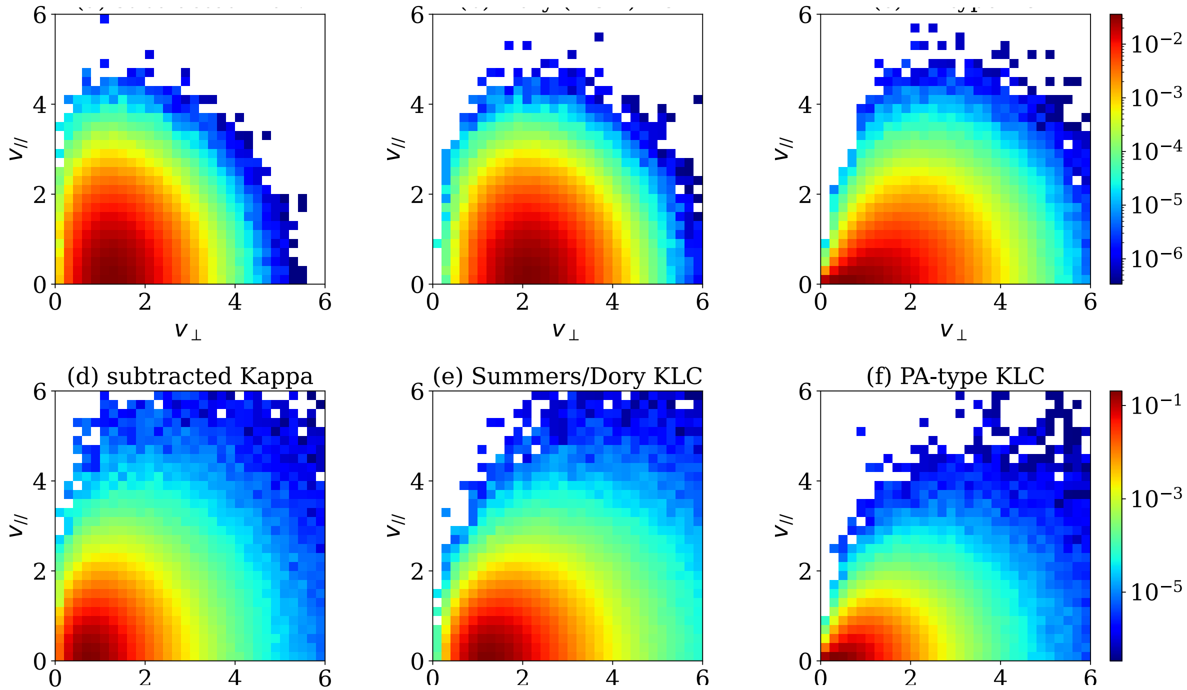
Loss-cone distributions



Subtracted Maxwellian

Dory-type loss-cone

Pitch-angle type loss-cone



Subtracted Kappa

Summers/Dory -type KLC

Pitch-angle type KLC

Subtracted Maxwellian

[Ashour-Abdalla & Kennel, 1978]

$$f(v_\parallel, v_\perp) = \frac{N_0}{\pi^{1/2}\theta_\parallel} \exp\left(-\frac{v_\parallel^2}{\theta_\parallel^2}\right) \times \boxed{\frac{1}{\pi\theta_\perp^2(1-\beta)} \left\{\exp\left(-\frac{v_\perp^2}{\theta_\perp^2}\right) - \exp\left(-\frac{v_\perp^2}{\beta\theta_\perp^2}\right)\right\}}$$

$$\boxed{x \equiv v_\perp^2/\theta_\perp^2 \qquad f_X(x) = \frac{1}{1-\beta}\left(\exp(-x) - \exp\left(-\frac{x}{\beta}\right)\right)}$$

- Convolution of two exponential distributions

$$x \leftarrow -\log U_1 - \beta \log U_2$$

$$\boxed{s \sim G_s(s) = \exp(-s), \quad s \geq 0}$$

$$\boxed{t \sim G_t(t) = \frac{1}{\beta}\exp\left(-\frac{t}{\beta}\right), \quad t \geq 0}$$

$$G(x) = \int_0^x G_s(s) \cdot G_t(x-s)\, ds = \frac{1}{\beta}\int_0^x \exp\left(-\frac{\beta-1}{\beta}s - \frac{x}{\beta}\right) ds$$
$$= \frac{1}{1-\beta}\left\{\exp(-x) - \exp\left(-\frac{x}{\beta}\right)\right\}$$

- Recipe

Algorithm for the Subtracted Maxwellian

Algorithm 2

generate $U_1, U_2, U_3 \sim U(0,1)$

generate $N \sim \mathcal{N}(0,1)$

$x \leftarrow -\log U_1 - \beta \log U_2$

$v_{\perp 1} \leftarrow \theta_\perp \sqrt{x}\, \cos(2\pi U_3)$

$v_{\perp 2} \leftarrow \theta_\perp \sqrt{x}\, \sin(2\pi U_3)$

$v_\parallel \leftarrow \theta_\parallel \sqrt{1/2}\;\; N$

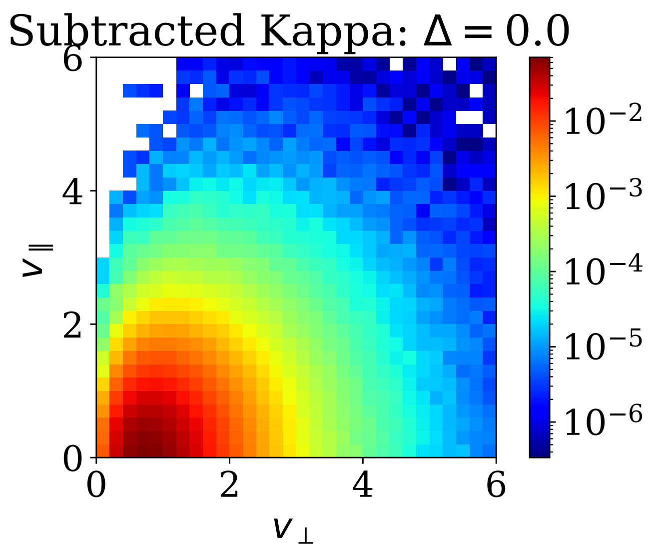
return $v_{\perp 1}, v_{\perp 2}, v_\parallel$

Subtracted Kappa distribution

[Summers & Stone, 2025]

$$f_{\rm sk}(v_\parallel, v_\perp)\, d^3v = N \cdot C_{\rm sk} \left\{ \frac{1-\Delta\beta}{1-\beta} \left(1 + \frac{{v_\parallel}^2}{\kappa\theta_\parallel^2} + \frac{v_\perp^2}{\kappa\theta_\perp^2} \right)^{-(\kappa+1)} - \frac{1-\Delta}{1-\beta} \left(1 + \frac{{v_\parallel}^2}{\kappa\theta_\parallel^2} + \frac{v_\perp^2}{\beta\kappa\theta_\perp^2} \right)^{-(\kappa+1)} \right\} d^3v$$

- Inspired by the t-generator, we divide sub-Max by Gamma



Algorithm for the Subtracted Kappa Distribution

Algorithm 5: subtracted Kappa ($\kappa > 3/2$)

generate $U_1, U_2, U_3 \sim U(0,1)$

generate $N \sim \mathcal{N}(0,1)$

generate $Y \sim \mathrm{Ga}(\kappa - 1/2, 2)$

$x \leftarrow -\log U_1 - \beta \log(\min(\frac{U_2}{1-\Delta}, 1))$

$v_{\perp 1} \leftarrow \theta_\perp \sqrt{2\kappa x} \cos(2\pi U_3)/\sqrt{Y}$

$v_{\perp 2} \leftarrow \theta_\perp \sqrt{2\kappa x} \sin(2\pi U_3)/\sqrt{Y}$

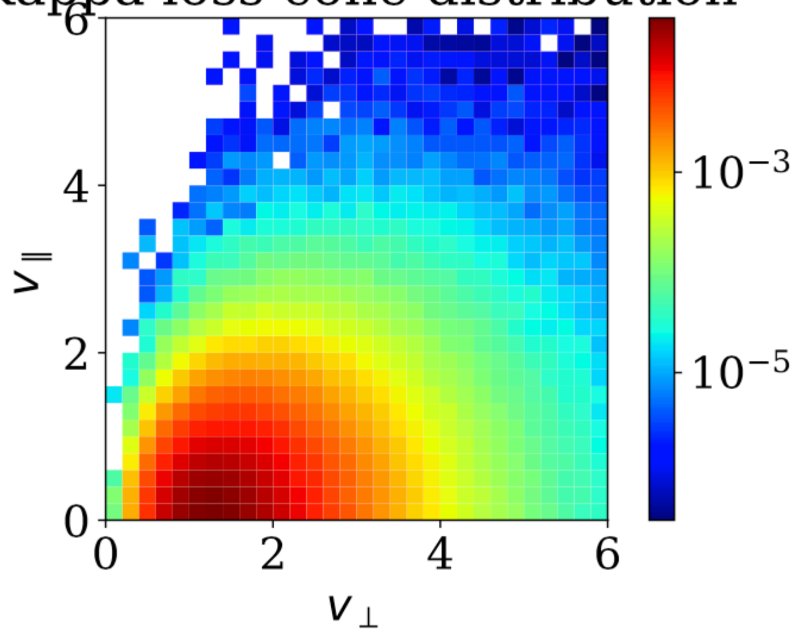
$v_\parallel \leftarrow \theta_\parallel \sqrt{\kappa} N/\sqrt{Y}$

return $v_{\perp 1}, v_{\perp 2}, v_\parallel$

SZ, Usami, & Matsukiyo 2026 JGR

Dory/Summers-type KLC distribution

$$f(\boldsymbol{v}) = \frac{N_0}{\pi^{3/2}\theta_\parallel \theta_\perp^2 \kappa^{j+3/2}} \frac{\Gamma(\kappa + j + 1)}{\Gamma(j+1)\Gamma(\kappa - 1/2)} \times \left(\frac{v_\perp}{\theta_\perp}\right)^{2j} \left(1 + \frac{{v_\parallel}^2}{\kappa\theta_\parallel^2} + \frac{{v_\perp}^2}{\kappa\theta_\perp^2}\right)^{-(\kappa+j+1)}$$



Algorithm 4

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generate $N \sim \mathcal{N}(0, 1)$

generate $Y \sim \mathrm{Ga}(\kappa - 1/2, 2)$ Gamma variate

generate $X \sim \mathrm{Ga}(j + 1, 2)$ Gamma variate

generate $U \sim U(0, 1)$

$v_{\perp 1} \leftarrow \theta_\perp \sqrt{\kappa X} \cos(2\pi U)/\sqrt{Y}$

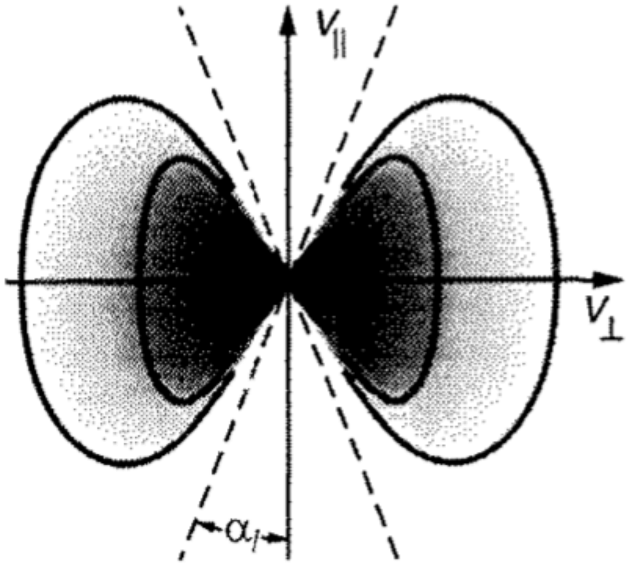
$v_{\perp 2} \leftarrow \theta_\perp \sqrt{\kappa X} \sin(2\pi U)/\sqrt{Y}$

$v_\parallel \leftarrow \theta_\parallel \sqrt{\kappa} N/\sqrt{Y}$

return $v_{\perp 1}, v_{\perp 2}, v_\parallel$

SZ & Nakano 2023 JGR

Pitch-angle (PA) type distribution



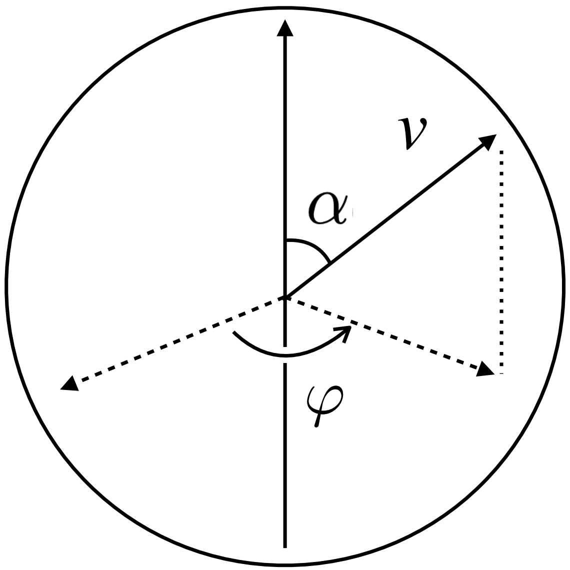
Loss−Cone Distribution

$$\propto \left(\sin\alpha \right)^{2j}$$

$$\Leftrightarrow \quad \propto \left(\frac{v_\perp}{v} \right)^{2j}$$

c.f. Kennel 1966

Utilizing Beta distribution



$$\iiint f_0(v)\ (\sin\alpha)^{2j}\, d^3 v$$

$$= 4\pi\left(\int_0^\infty v^2 f_0(v)\ dv\right)\left(\int_0^{\pi/2} (\sin\alpha)^{2j+1}\, d\alpha\right)$$

$$x \equiv \cos^2\alpha$$

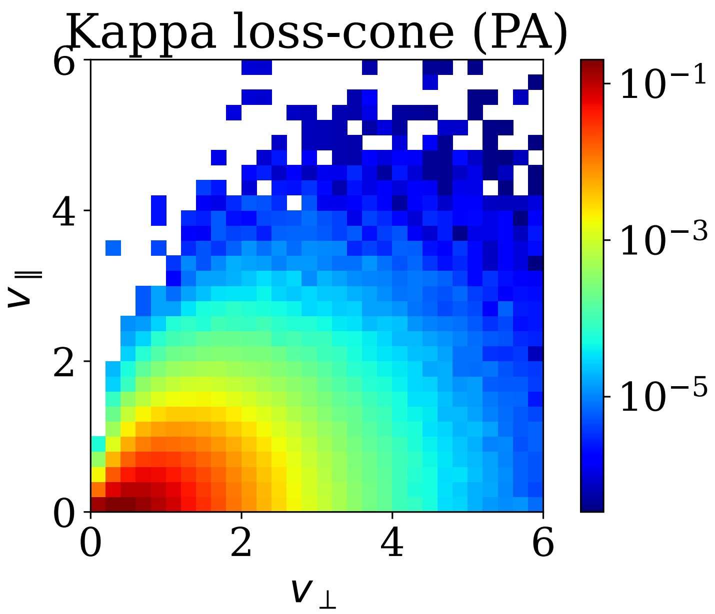
$$\longrightarrow \frac{B(1/2, j+1)}{2}\left\{\int_0^1 \frac{(1-x)^j x^{-1/2}}{B(1/2, j+1)} dx\right\}$$

Beta distribution

- One can transform isotropic distributions to loss-cone distributions via Beta random variate.

Pitch-angle-type KLC distribution

$$f(v) = \frac{N_0}{\pi^2 \theta^3 \kappa^{3/2}} \frac{2\Gamma(j+3/2)\Gamma(\kappa+1)}{\Gamma(j+1)\Gamma(\kappa-1/2)} \times \left(\frac{v_\perp}{v}\right)^{2j} \left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}$$



Algorithm 5.4: KLC distribution

generate $N \sim \mathcal{N}(0,1)$
generate $Y \sim \mathrm{Ga}(\kappa - 1/2, 2)$
generate $X_1 \sim \mathrm{Ga}(3/2, 2)$
generate $X_2 \sim \mathrm{Ga}(j+1, 2)$
generate $U \sim U(0,1)$

$$v_{\perp 1} \leftarrow \theta \sqrt{\frac{\kappa X_1}{Y}} \sqrt{\frac{X_2}{N^2+X_2}} \cos(2\pi U)$$

$$v_{\perp 2} \leftarrow \theta \sqrt{\frac{\kappa X_1}{Y}} \sqrt{\frac{X_2}{N^2+X_2}} \sin(2\pi U)$$

$$v_{\parallel} \leftarrow \theta \sqrt{\frac{\kappa X_1}{Y}} \frac{N}{\sqrt{N^2+X_2}}$$

return $v_{\perp 1}, v_{\perp 2}, v_{\parallel}$

Loss-cone transform (Beta variate)

Three elements

Table 3
Algorithm for the Summers-Type Kappa

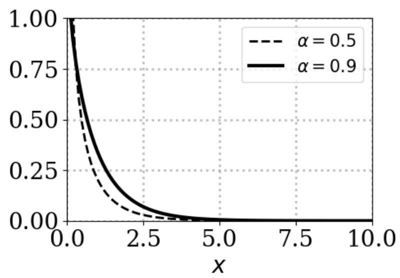
Algorithm 4
generate $N \sim \mathcal{N}(0, 1)$
generate $Y \sim \mathrm{Ga}(\kappa - 1/2, 2)$
generate $X \sim \mathrm{Ga}(j + 1, 2)$
generate $U \sim U(0, 1)$
$v_{\perp 1} \leftarrow \theta_{\perp} \sqrt{\kappa X} \cos(2\pi U) / \sqrt{Y}$
$v_{\perp 2} \leftarrow \theta_{\perp} \sqrt{\kappa X} \sin(2\pi U) / \sqrt{Y}$
$v_{\parallel} \leftarrow \theta_{\parallel} \sqrt{\kappa} N / \sqrt{Y}$
return $v_{\perp 1}, v_{\perp 2}, v_{\parallel}$

- 1. Uniform random number
  - $0 \leq x < 1$
  - rand()
- 2. Normal random number
  - Gaussian
  - randn(), Box-Muller, ziggurat...
- 3. Gamma random number
  - Gamma distribution
  - gamma()??

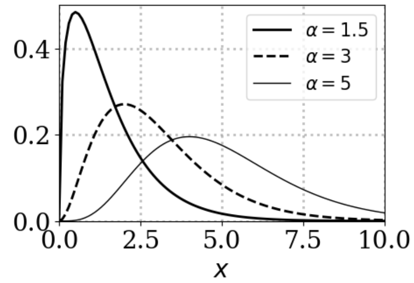
Gamma-distributed random number

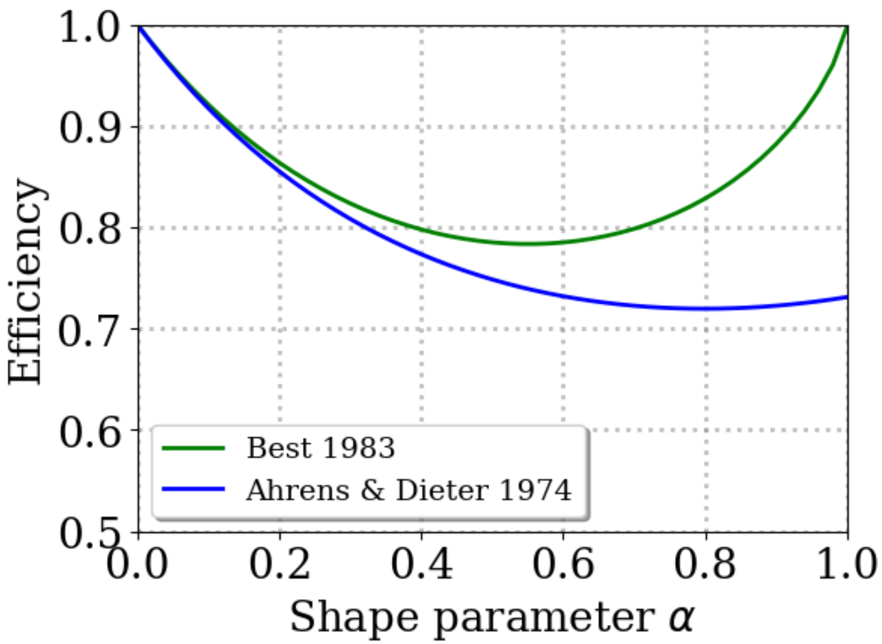
$\alpha<1$

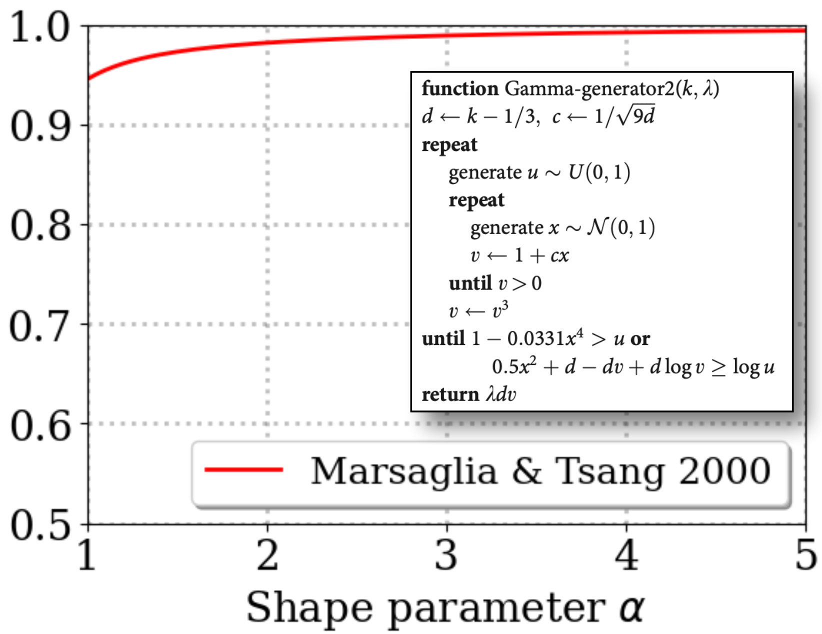
$1<\alpha$



$$\mathrm{Ga}(x;\alpha,1)=\frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}$$

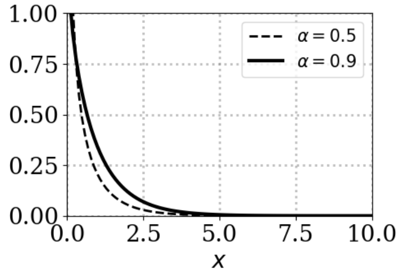




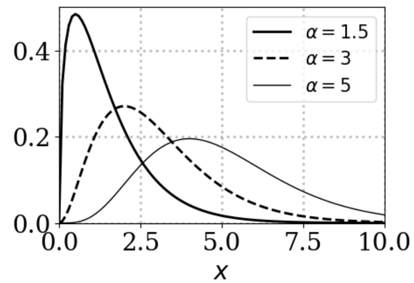


Gamma-distributed random number

$\alpha<1$

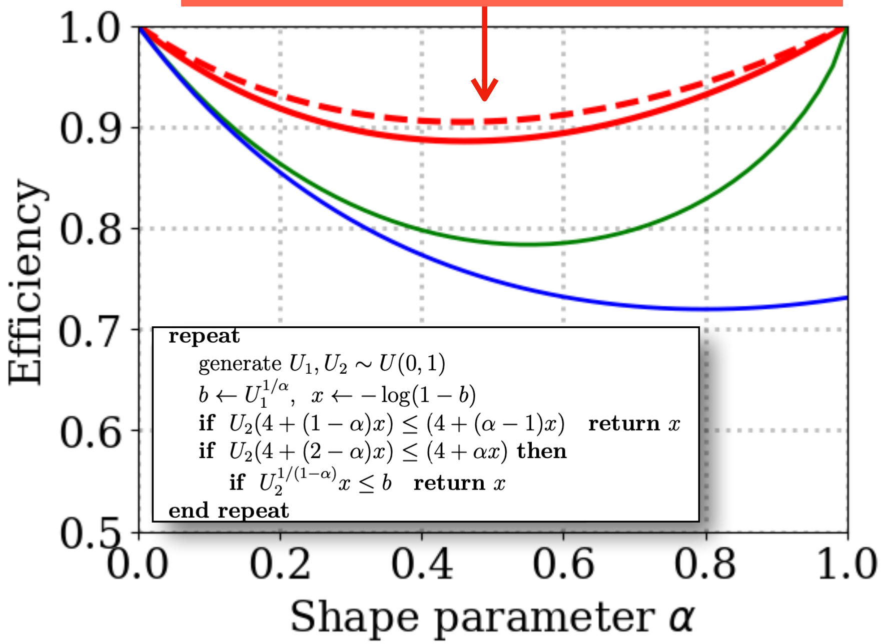


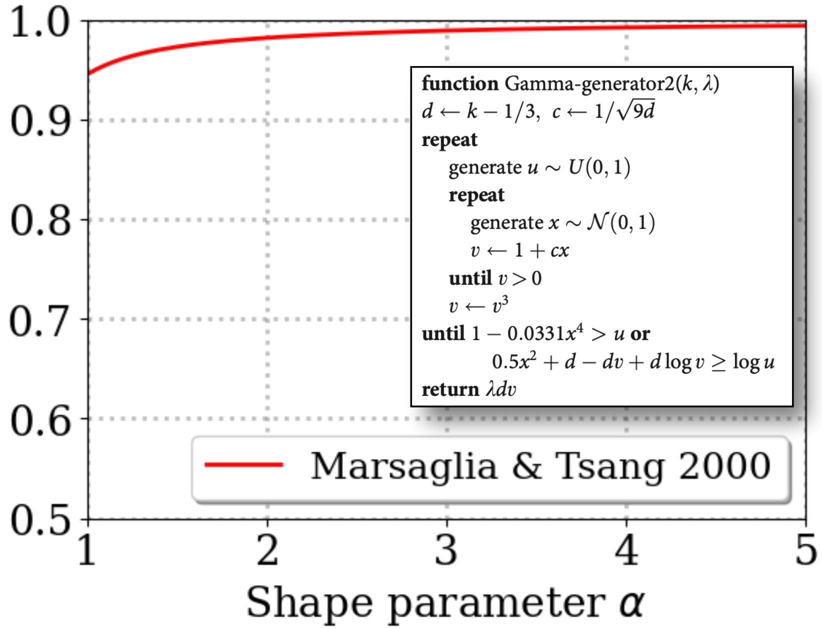
$1<\alpha$



$$\mathrm{Ga}(x;\alpha,1)=\frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}$$

Zenitani 2024, *Economics Bulletin*





Summary

Based on three elements:

1. Uniform
2. Normal
3. Gamma

- 1. Kappa distribution [1-3]
  - t-generator, Pareto method, Approximate generator
- 2. Relativistic Kappa distribution [1]
  - Superpositions of four gamma distributions
- 3. Kappa loss-cone distributions [4,5]
  - Subtracted methods, scattering by Beta distribution
- 4. Gamma distribution [6]
  - New generator for shape parameter less than unity

- References

- [1] Zenitani & Nakano, *Phys. Plasmas*, 2022
- [2] Zenitani, *RNAAS*, 2025 (arxiv:2512.04272)
- [3] Zenitani & Umeda, *EPS*, 2026
- [4] Zenitani & Nakano, *JGR Space Phys.*, 2023
- [5] Zenitani et al., *JGR Space Phys.*, 2026
- [6] Zenitani, *Econ. Bull.*, 2024