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Electromagnetic Particle-In-Cell Model With Adaptive Mesh Refinement

Keizo Fujimoto[1] and Richard Sydora[2] (*E-mail*: keizo@stelab.nagoya-u.ac.jp)

1. Solar-Terrestrial Environment Laboratory, Nagoya University, Nagoya, Aichi 464-8601, Japan

2. Department of Physics, University of Alberta, Edmonton, Alberta T6G 2G7, Canada

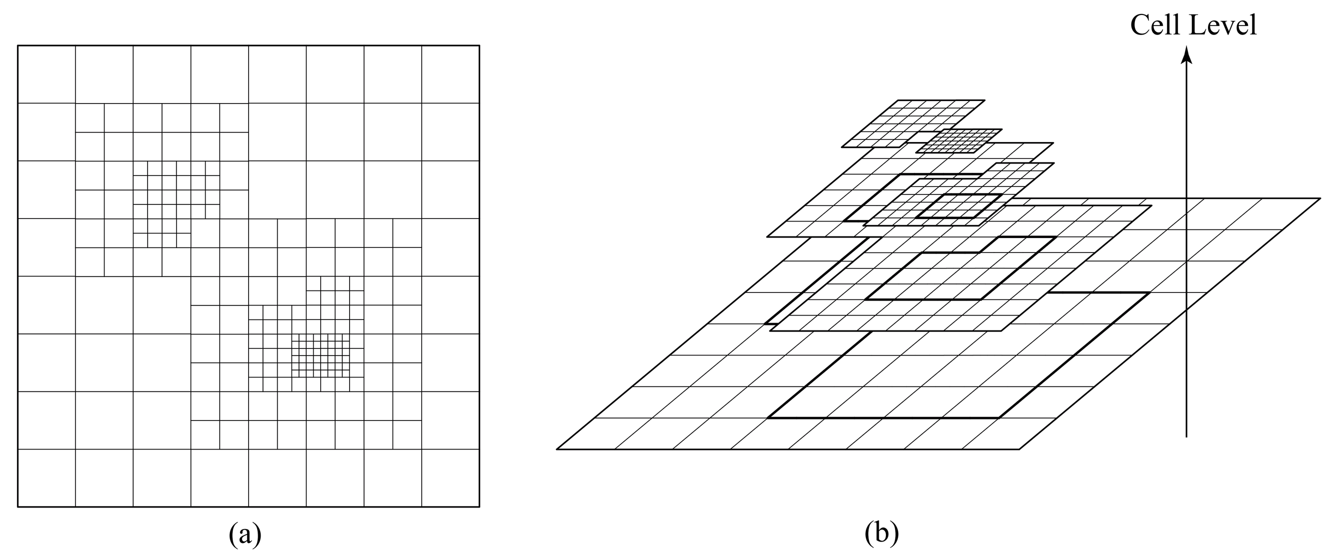
1. Introduction

Local kinetic processes in space and laboratory plasmas are, in many cases, considered to have a significant impact on global-scale energy transport and conversion processes. Magnetic reconnection is a typical case in which microscopic processes in the diffusion region, formed around the magnetic X-line, can control the MHD-scale dynamics. Generally, it is very difficult to describe in numerical approach such a phenomenon that includes a number of physical scales which can couple with each other.

The adaptive mesh refinement (AMR) technique is one of the promising methods to overcome the difficulties. It subdivides the computational cells locally in space and dynamically in time. In this study, we apply the AMR to the electromagnetic particle-in-cell (EM-PIC) model for both the 2D and 3D systems, and successfully achieve efficient high-resolution simulations on the nonlinear evolution of the plasma sheet.

2. Data Structure

The AMR technique subdivides only cells that satisfy some refinement criteria and enhances the local spatial resolution. The data sets for the child cells are added onto the parent cell and develop a hierarchical tree structure.



The hierarchical cell structure is supported by a set of pointers.

Oct pointers

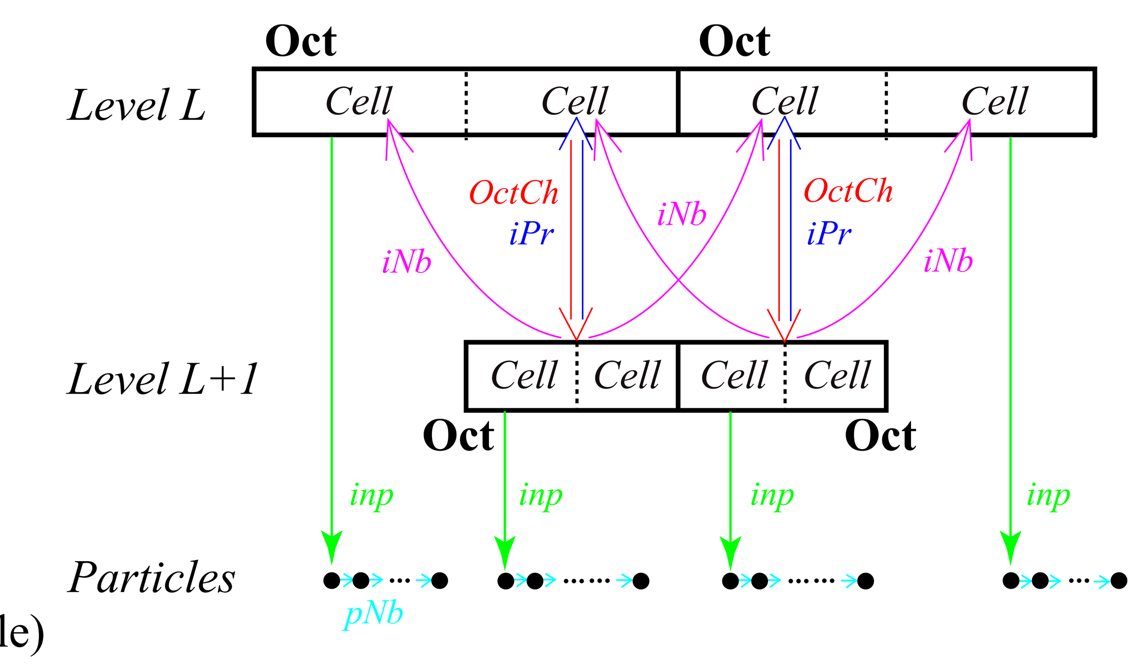
- iPr (to parent cell)
- iNb (to parent cell of the neighboring Oct)
- inp (to a particle belonging to Oct)

Cell pointers

- OctCh (to child Oct)

Particle pointers

- pNb (to a neighboring particle)



3. Particle Splitting & Coalescence

The number of particles per cell is controlled by splitting particles in fine cells and coalescing them in coarse cells.

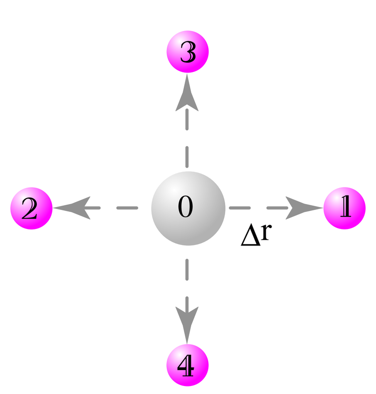
(a) Splitting in 2D

$\Delta r = \Delta_L/N_p^{1/2}$

$\boldsymbol{v}_{1,2,3,4} = \boldsymbol{v}_0$

$m_{1,2,3,4} = m_0/4$

$q_{1,2,3,4} = q_0/4$

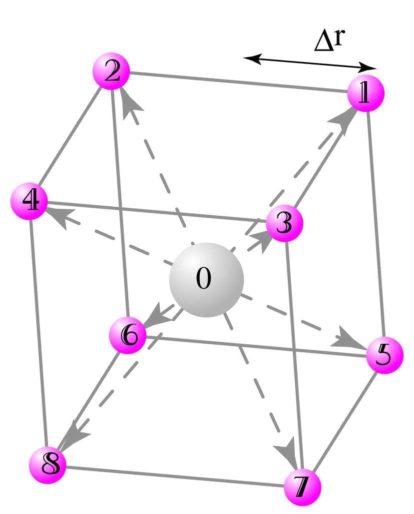


(b) Splitting in 3D

$\Delta r = \Delta_L/N_p^{1/3}$

$\boldsymbol{v}_{1,\ldots,8} = \boldsymbol{v}_0$

$m_{1,\ldots,8} = m_0/8$; $q_{1,\ldots,8} = q_0/8$



(c) Coalescence

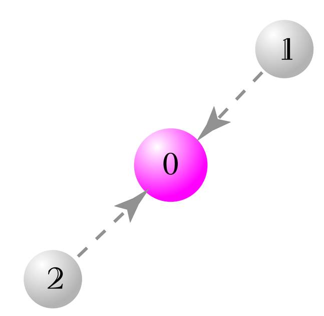
$m_1 = m_2$; $q_1 = q_2$

$\boldsymbol{x}_0 = (\boldsymbol{x}_1 + \boldsymbol{x}_2)/2$

$\boldsymbol{v}_0 = (\boldsymbol{v}_1 + \boldsymbol{v}_2)/2$

$m_0 = m_1 + m_2$

$q_0 = q_1 + q_2$



4. Basic Equations

Particle Motions

$$\frac{\boldsymbol{v}^{n+1/2} - \boldsymbol{v}^{n-1/2}}{\Delta t} = \frac{q}{m}\left[\boldsymbol{E}^n(\boldsymbol{x}^n) + \frac{\boldsymbol{v}^{n+1/2} + \boldsymbol{v}^{n-1/2}}{2} \times \boldsymbol{B}^n(\boldsymbol{x}^n)\right]$$

$$\frac{\boldsymbol{x}^{n+1} - \boldsymbol{x}^n}{\Delta t} = \boldsymbol{v}^{n+1/2}$$

Electric & Magnetic Field

$$\boldsymbol{E}_L = -\nabla\phi;\quad \nabla^2\phi = -\rho/\varepsilon_0$$

$$\frac{\boldsymbol{E}_T^{n+1} - \boldsymbol{E}_T^n}{\Delta t} = c^2\nabla\times\frac{\boldsymbol{B}^{n+1} + \boldsymbol{B}^n}{2} - \boldsymbol{j}_T^{n+1/2}/\varepsilon_0;\quad \boldsymbol{j}_T^{n+1/2} = \boldsymbol{j}^{n+1/2} + \varepsilon_0\frac{\boldsymbol{E}_L^{n+1} - \boldsymbol{E}_L^n}{\Delta t}$$

$$\frac{\boldsymbol{B}^{n+1} - \boldsymbol{B}^n}{\Delta t} = -\nabla\times\frac{\boldsymbol{E}_T^{n+1} + \boldsymbol{E}_T^n}{2}$$

$$\boldsymbol{E}^{n+1} = \boldsymbol{E}_L^{n+1} + \boldsymbol{E}_T^{n+1}$$

5. Evolution of the Plasma Sheet

$B_x(z) = -B_0\tanh(z/\lambda)$; $n(z) = n_{ps}\,\mathrm{sech}^2(z/\lambda) + n_b\tanh^2(z/\lambda)$

$m_i/m_e = 100$; $T_{i,ps}/T_{e,ps} = 5.0$; $c/V_A = 20$

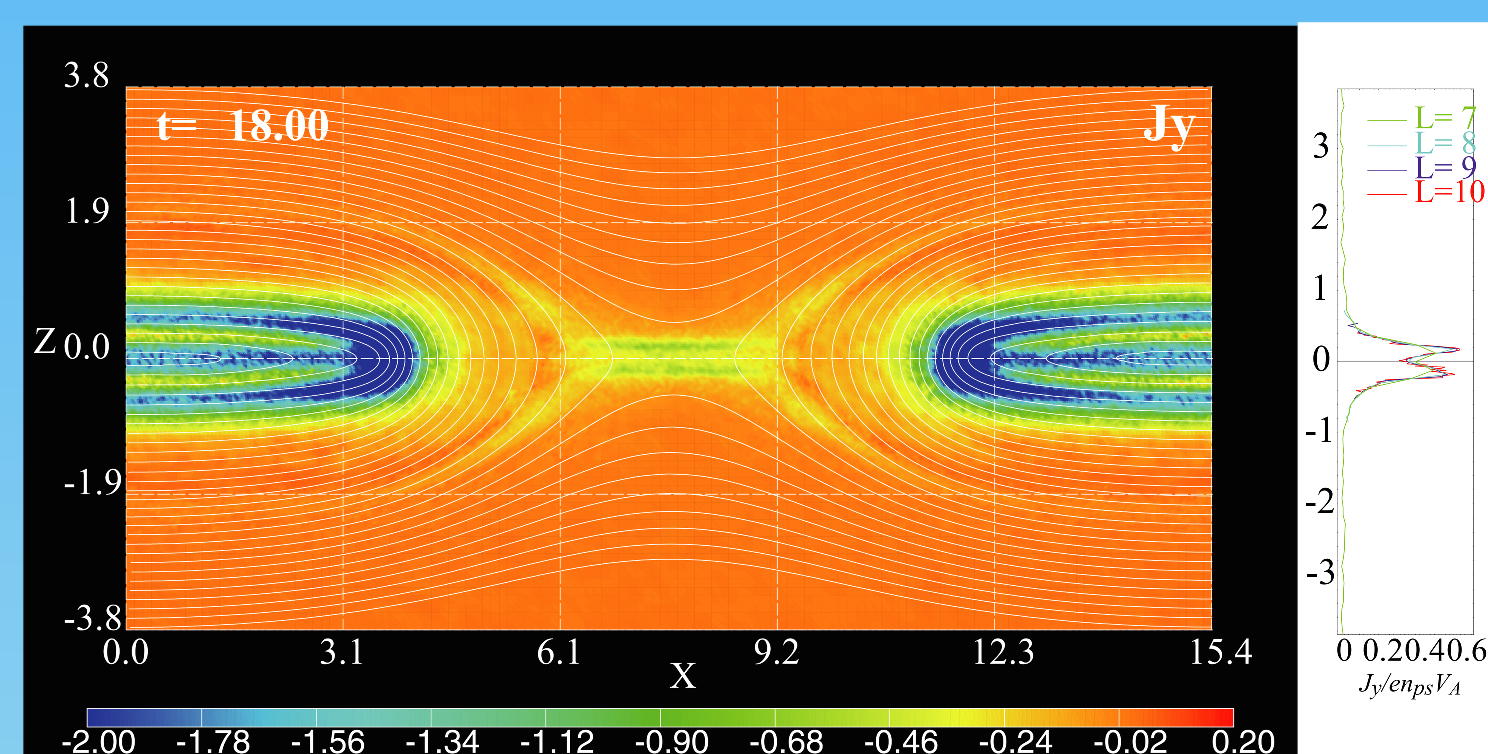
$\lambda = 0.5\,\lambda_i$; $n_b = 0.044\,n_{ps}$

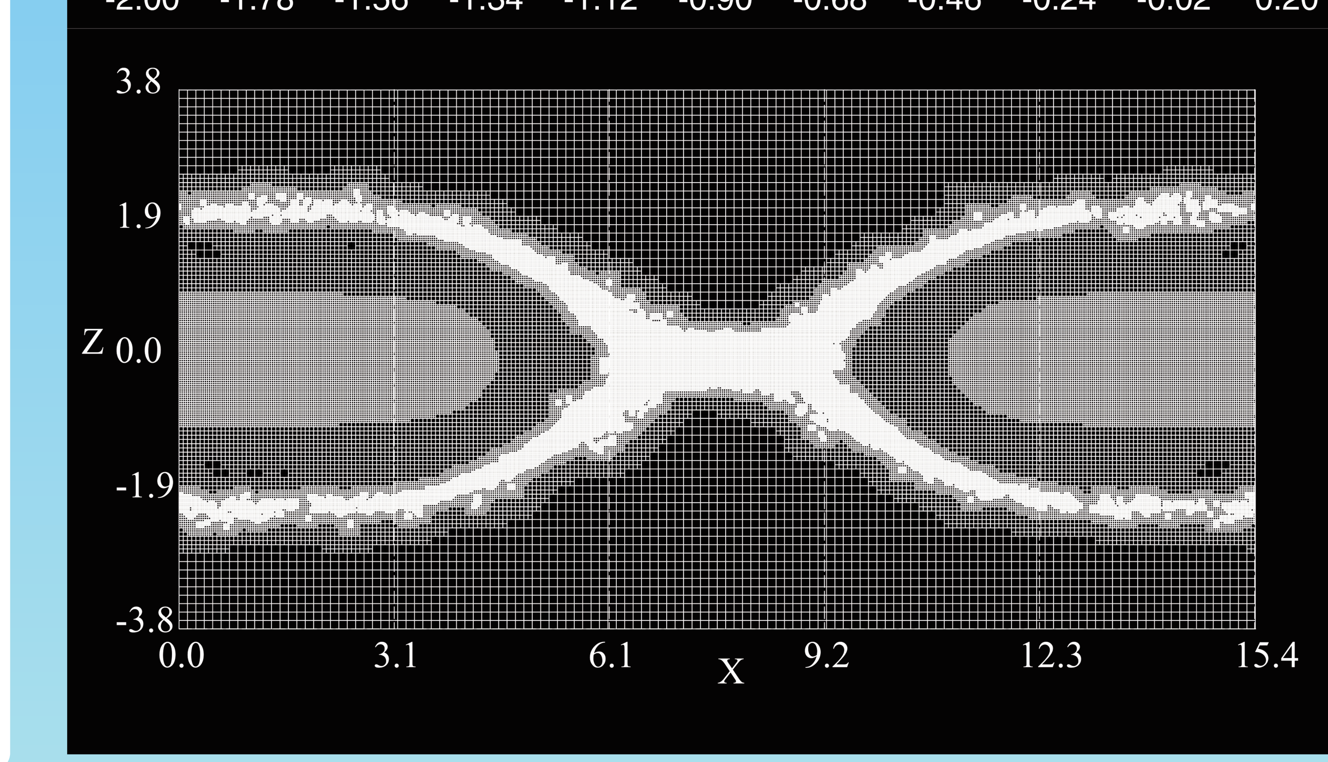
Refinement Criteria

$\Delta_L > 2.0\,\lambda_{De}$ or $V_{ey} > 2.0\,V_A$

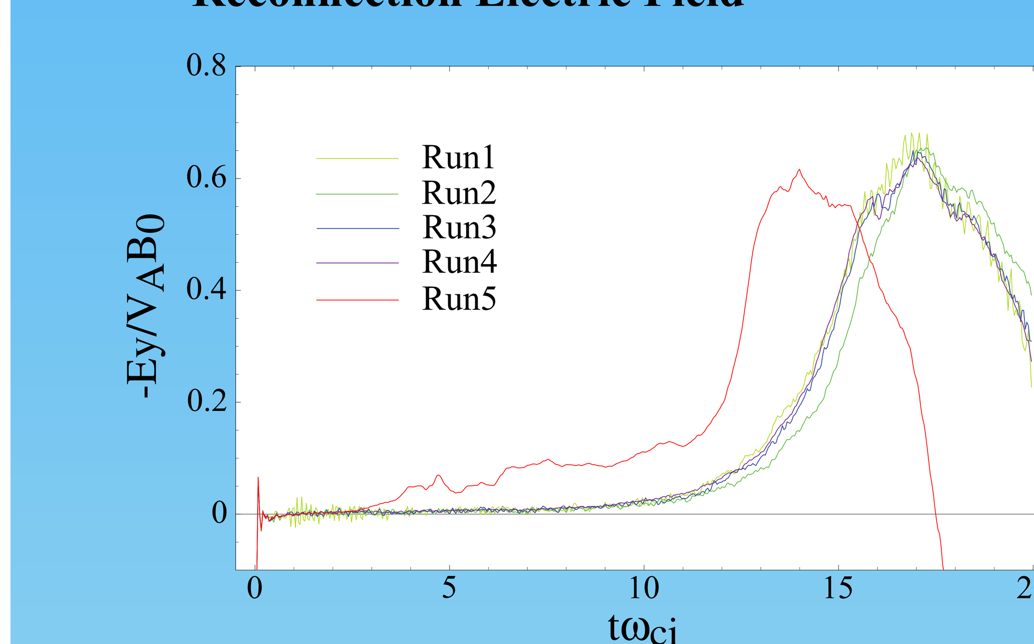
Simulation information.

Run#	AMR	Particle splitting	System size	N_{ct}	N_{pt}
Run1	No	No	15.4 ×15.4	1.0×10^6	1.9×10^7
Run2	Yes	No	15.4×15.4	6.7×10^4	1.9×10^7
Run3	Yes	Yes	15.4 ×15.4	7.1×10^4	4.9×10^6
Run4	Yes	Yes	15.4 ×0.24× 15.4	5.0×10^5	4.8×10^7
Run5	Yes	Yes	15.4 ×1.92× 15.4	3.3×10^6	5.4×10^8

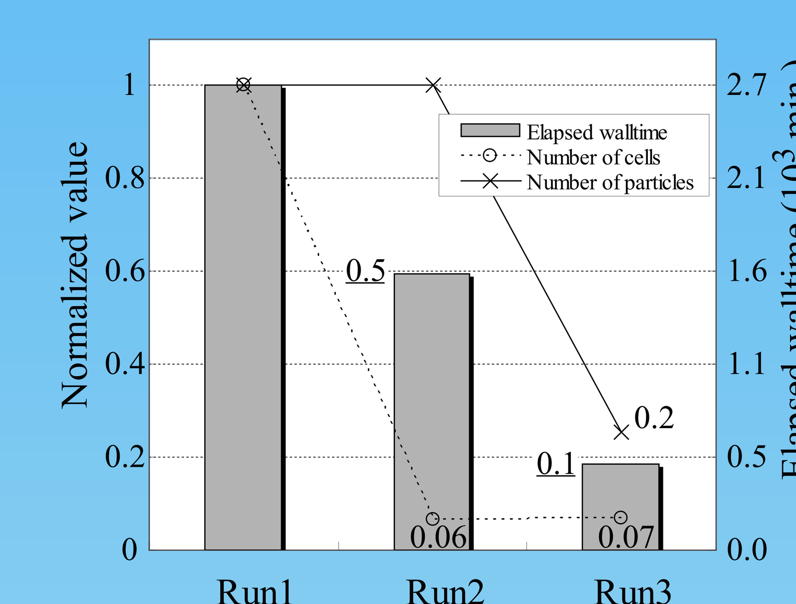




Reconnection Electric Field



Simulation Time



6. Summary

We have successfully developed a new EM-PIC code with the AMR both in the 2D and 3D systems. In order to control the number of particles per cell, the present code also performs the particle splitting and coalescence.

We have demonstrated that the AMR and particle splitting-coalescence techniques, combined with the EM-PIC code, enable efficient high-resolution simulations of the plasma sheet, and can be a promising method for studying physical phenomena which include a number of physical scales that can be coupled with each other.

Ref. K. Fujimoto, and S. Machida, *J. Comput. Phys.*, 214 (2006) 550.

K. Fujimoto, and R. Sydora, *Comput. Phys. Comm.*, 178 (2008) 915.