Jeans Instability of Filamentary Clouds Threaded by Vertical Magnetic Fields

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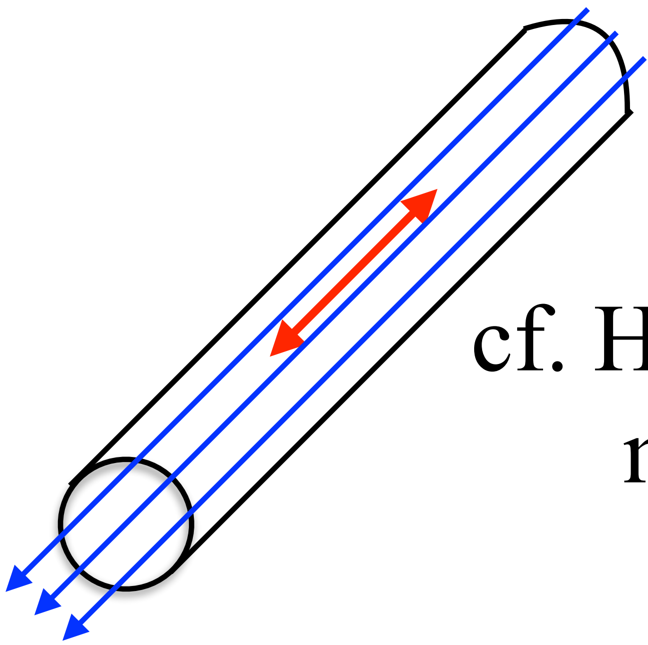
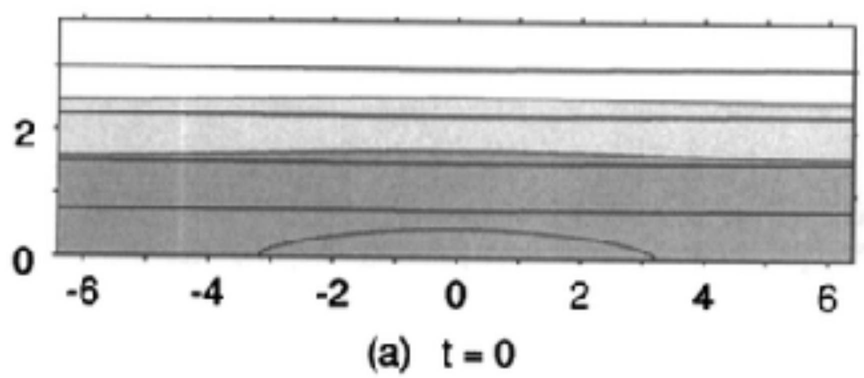
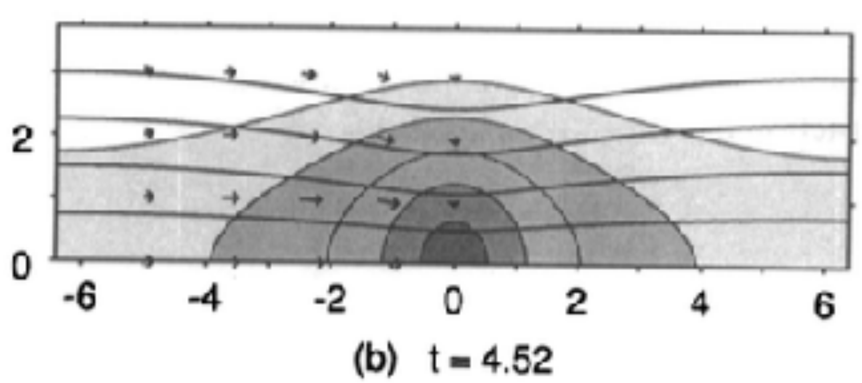
Clumps form filamentary clouds through fragmentation

longitudinal magnetic field



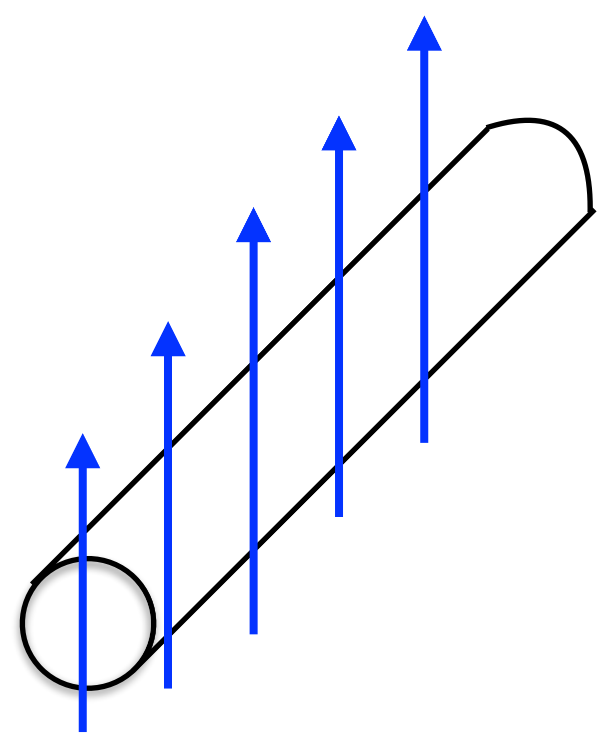
cf. Hanawa+ 92 and
many others





cf. Nakamura+ 93, 95

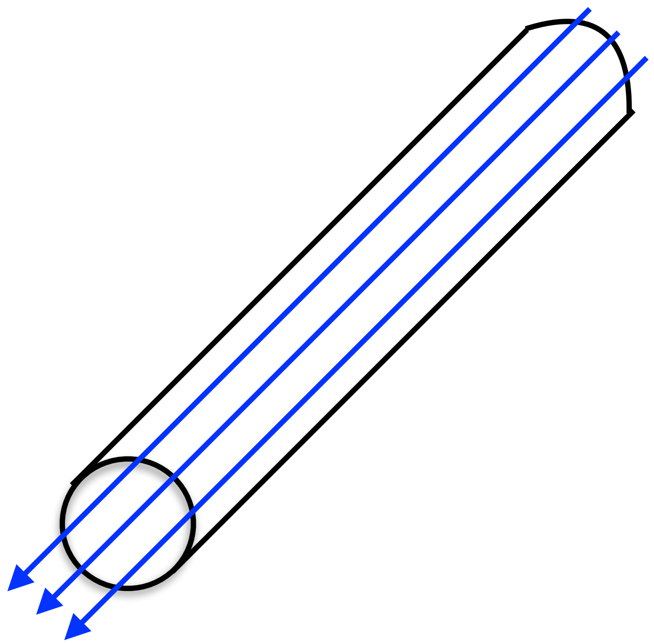
vertical magnetic field



?

this work

Equilibrium model:
Longitudinal Magnetic Field



1D

symmetric around the axis

$$\rho(r) = \rho_0 \left(1 + \frac{r^2}{8H^2}\right)^{-2}$$

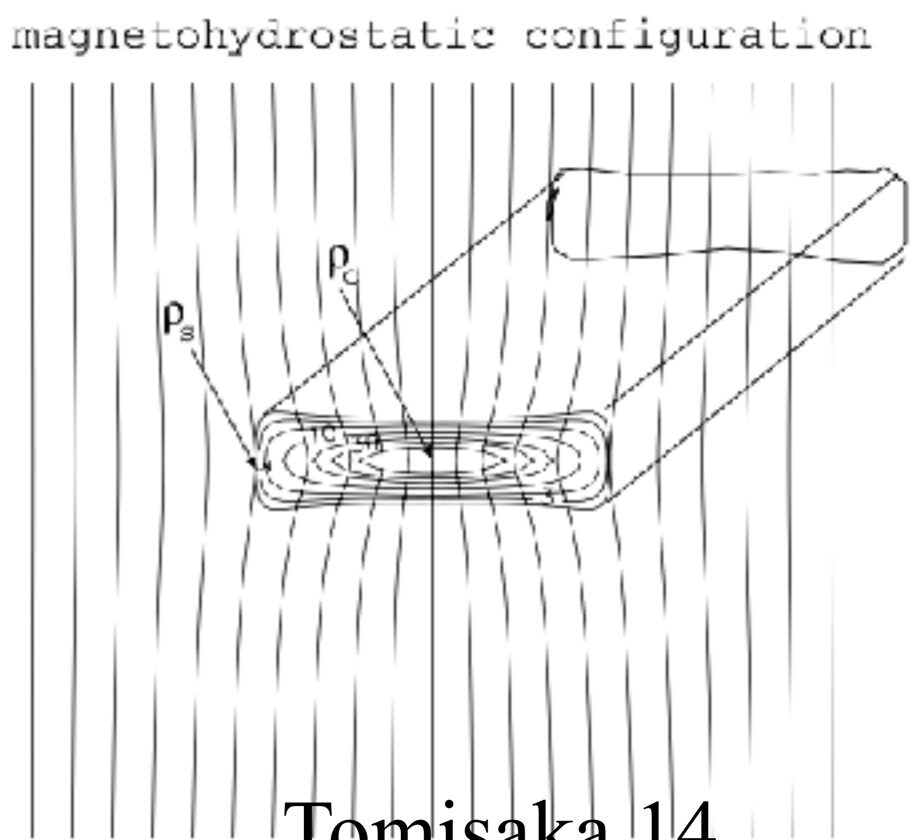
$$B_z(r) = B_0 \left(1 + \frac{r^2}{8H^2}\right)^{-1}$$

$$4\pi G \rho_0 H^2 = c_s^2 + \frac{B_0^2}{8\pi\rho_0}$$

supported in part by magnetic fields.
Stodolkiewicz 63

B_φ: hoop stress
cf. Fiege & Pudritz 00

Vertical Magnetic Field



Tomisaka 14

2D Flattened

Idealized Equilibrium Model

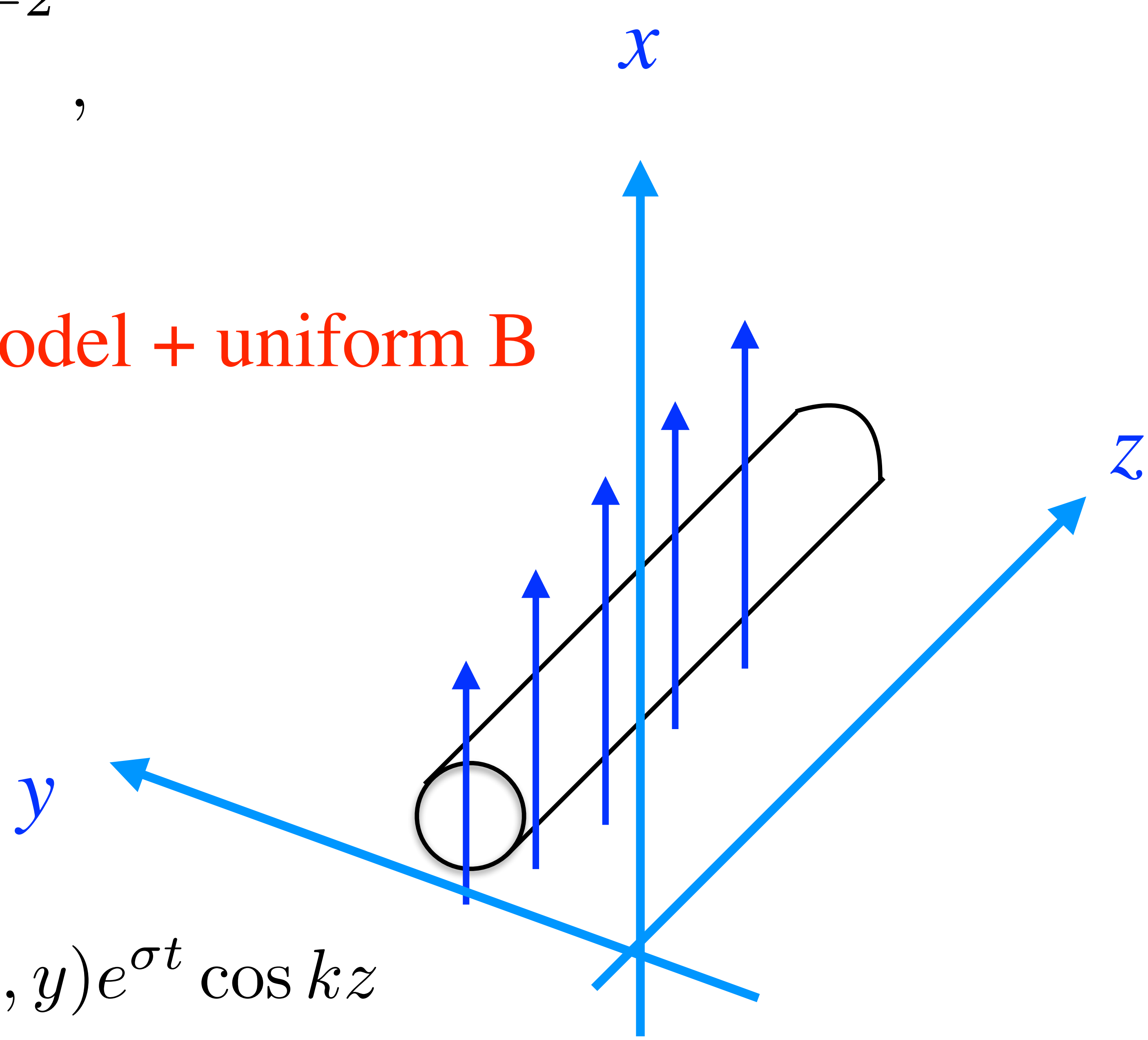
$$\rho_0 = \rho_c \left(1 + \frac{x^2 + y^2}{8H^2}\right)^{-2},$$

$$H^2 = \frac{c_s^2}{4\pi G \rho_c},$$

$$\boldsymbol{B}_0 = B_0 \boldsymbol{e}_x,$$

1D model + uniform B

isothermal cloud



3D perturbation

$$\rho(x, y, z, t) = \rho_0(x, y) + \varrho(x, y) e^{\sigma t} \cos kz$$

$$\beta = \frac{8\pi \rho_c c_s^2}{B_0^2}$$

Equilibrium

$$\rho_0 = \rho_c \left(1 + \frac{x^2 + y^2}{8H^2}\right)^{-2},$$

$$H^2 = \frac{c_s^2}{4\pi G \rho_c},$$

$$\boldsymbol{B}_0 = B_0 \boldsymbol{e}_x,$$

$\boldsymbol{x}$: magnetic field, $\boldsymbol{z}$: filament axis

$\boldsymbol{c}_{\mathrm{s}}$: sound speed

Ideal MHD Eq.

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}),$$

$$\frac{d\boldsymbol{v}}{dt} = -c_s^2 \boldsymbol{\nabla} \ln \rho - \boldsymbol{\nabla} \psi + \boldsymbol{j} \times \boldsymbol{B},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} (\boldsymbol{v} \times \boldsymbol{B}),$$

$$\boldsymbol{j} = \frac{\boldsymbol{\nabla} \times \boldsymbol{B}}{4\pi},$$

$$\Delta \psi = 4\pi G \rho.$$

$$\rho = \rho_0 + e^{\sigma t} \varrho(x, y) \cos kz,$$

$$\boldsymbol{\xi} = e^{\sigma t} \left(\xi_x \cos kz \boldsymbol{e}_x + \xi_y \cos kz \boldsymbol{e}_y + \xi_z \sin kz \boldsymbol{e}_z\right),$$

$$\boldsymbol{B} = \boldsymbol{B}_0 + e^{\sigma t} \left(b_x \cos kz \boldsymbol{e}_x + b_y \cos kz \boldsymbol{e}_y + b_z \sin kz \boldsymbol{e}_z\right),$$

$$\boldsymbol{J} = e^{\sigma t} \left(j_x \sin kz \boldsymbol{e}_x + j_y \sin kz \boldsymbol{e}_y + j_z \cos kz \boldsymbol{e}_z\right),$$

$$\psi = \psi_0 + e^{\sigma t} \delta\psi(x, y)$$

Numerical Methods

Displacement vector

$$\boldsymbol{\xi} = \int \boldsymbol{v} dt$$

$$\delta\varrho = -\frac{\partial}{\partial x}\left(\rho_0 \xi_x\right) - \frac{\partial}{\partial y}\left(\rho_0 \xi_y\right) - k\rho_0\xi_z,$$

$$b_x = -B_0\left[\frac{\partial}{\partial y}\xi_y(x,y) + k\xi_z\right],$$

$$b_y = B_0\frac{\partial \xi_y}{\partial x},$$

$$b_z = -B_0\frac{\partial \xi_z}{\partial x},$$

$$j_x = \frac{1}{4\pi}\left(\frac{\partial b_z}{\partial y} + kb_y\right),$$

$$j_y = -\frac{1}{4\pi}\left(k\delta b_x + \frac{\partial b_z}{\partial x}\right),$$

$$j_z = \frac{1}{4\pi}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right).$$

$$\delta\psi(\boldsymbol{r}) = \int \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\varrho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho_0\frac{d^2\boldsymbol{\xi}}{dt^2} = \boldsymbol{F}\left(\boldsymbol{\xi}\right),$$

$$\rho_0\sigma^2\boldsymbol{\xi} = \left(\boldsymbol{A} + \frac{B_0^2}{4\pi}\boldsymbol{C}\right)\boldsymbol{\xi}.$$

Force is proportional to ξ.

generalized eigenvalue problem

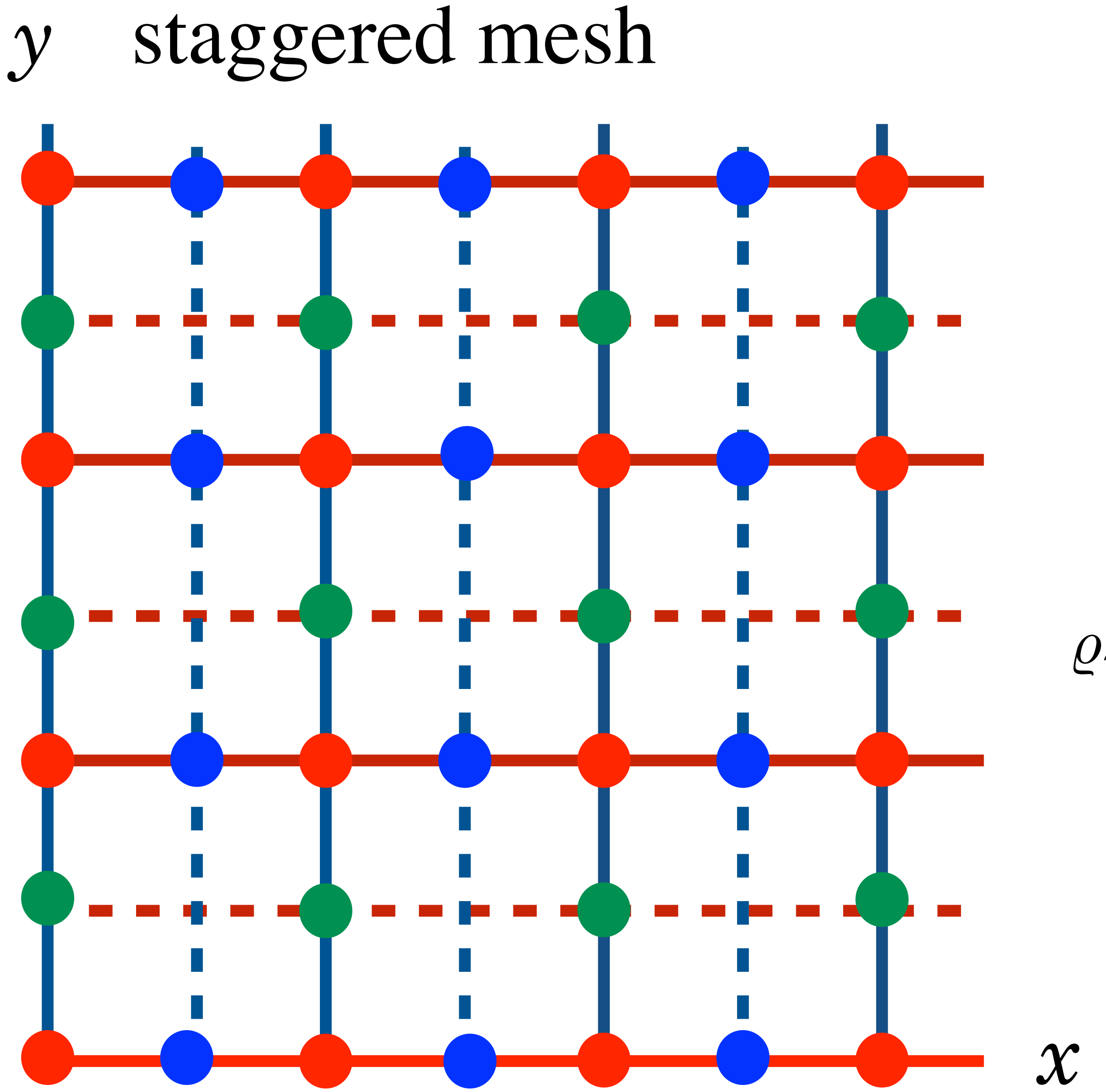
$$\left|\boldsymbol{A} + \frac{B_0^2}{4\pi}\boldsymbol{C} - \rho_0\boldsymbol{I}\right| = 0$$

LAPACK
Numerical Library

A perturbed quantity is expressed as a function ofξ.

Finite Difference Eq.

staggered mesh



- (red) $\xi_z,\ \varrho,\ \delta\psi,\ b_x,\ j_y$ — x, y sym
- (blue) $\xi_x,\ b_z$ — x anti, y sym
- (green) $\xi_y,\ j_z$ — x sym, y anti

$$\varrho_{i,j} = \frac{\rho_{0,i+1/2,j}\xi_{i+1/2,j} - \rho_{0,i-1/2,j}\xi_{i-1/2,j}}{\Delta x} - \frac{\rho_{0,i,j+1/2}\xi_{i,j+1/2} - \rho_{0,i,j-1/2}\xi_{i,j-1/2}}{\Delta y} - k\rho_{0,i,j}\xi_{z,i,j}$$

2nd order accuracy

Boundary (1) Fixed $\xi_x, \xi_y, \xi_z = 0$ for $x > n_x\Delta x$ or $y > n_y\Delta y$

(2) Free $\frac{\partial \boldsymbol{\xi}}{\partial x} = 0$ for $x > n_x\Delta x$

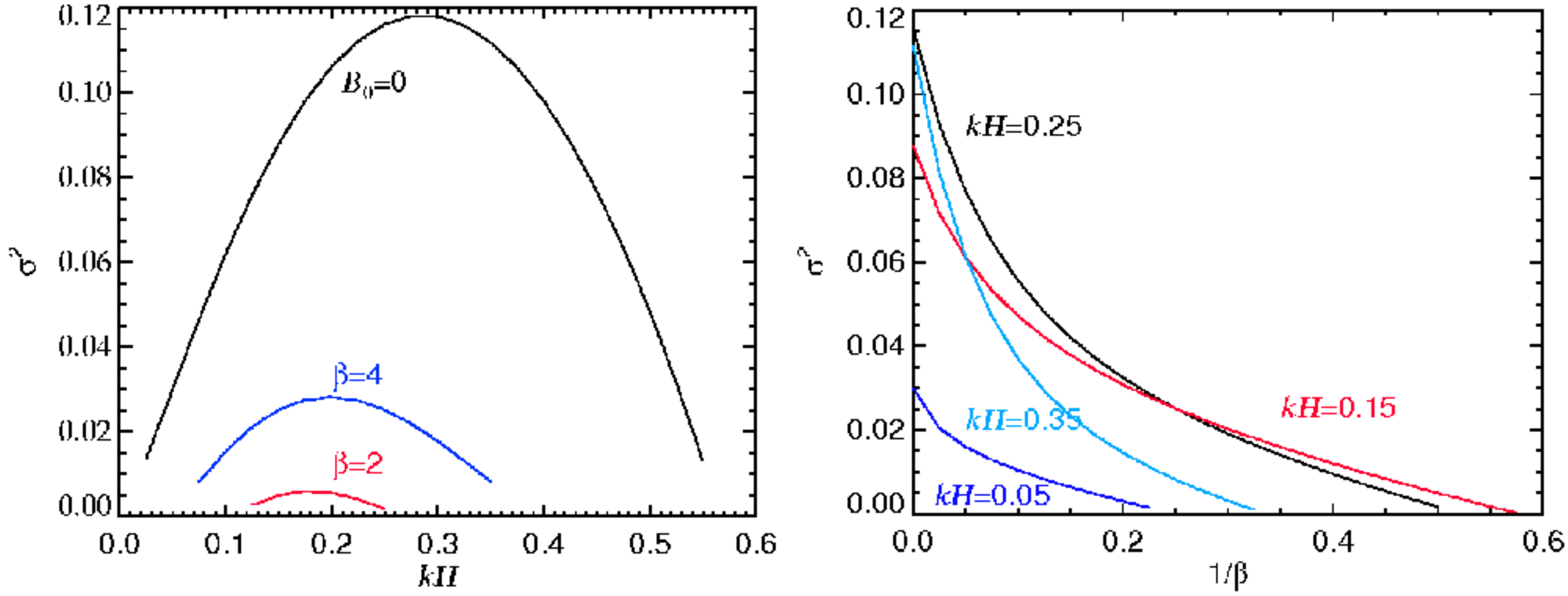
$\frac{\partial \boldsymbol{\xi}}{\partial y} = 0$ for $y > n_y\Delta y$

Fixed Boundary

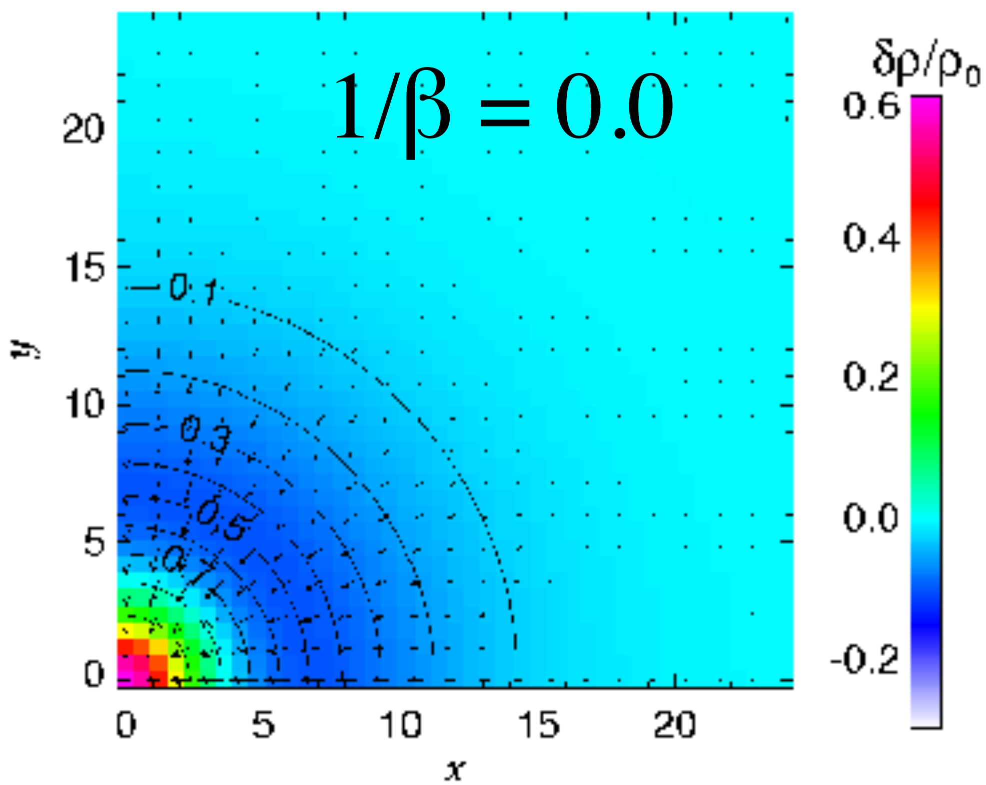
Growth rate

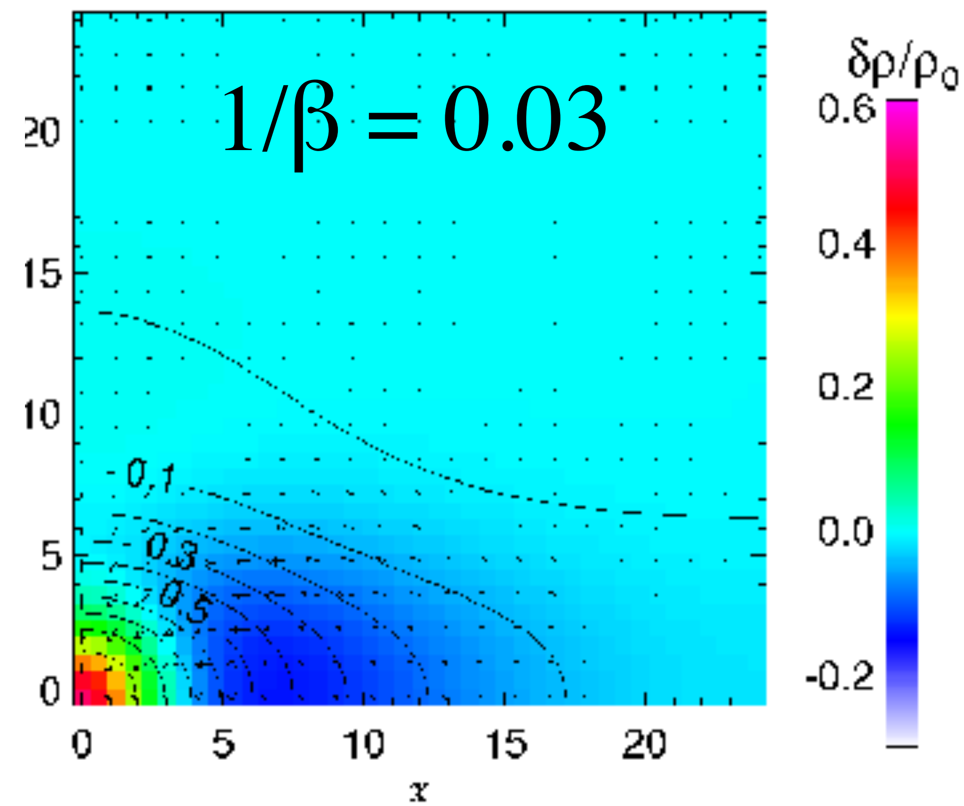
$$\frac{\sigma^2}{4\pi G \rho_c}$$

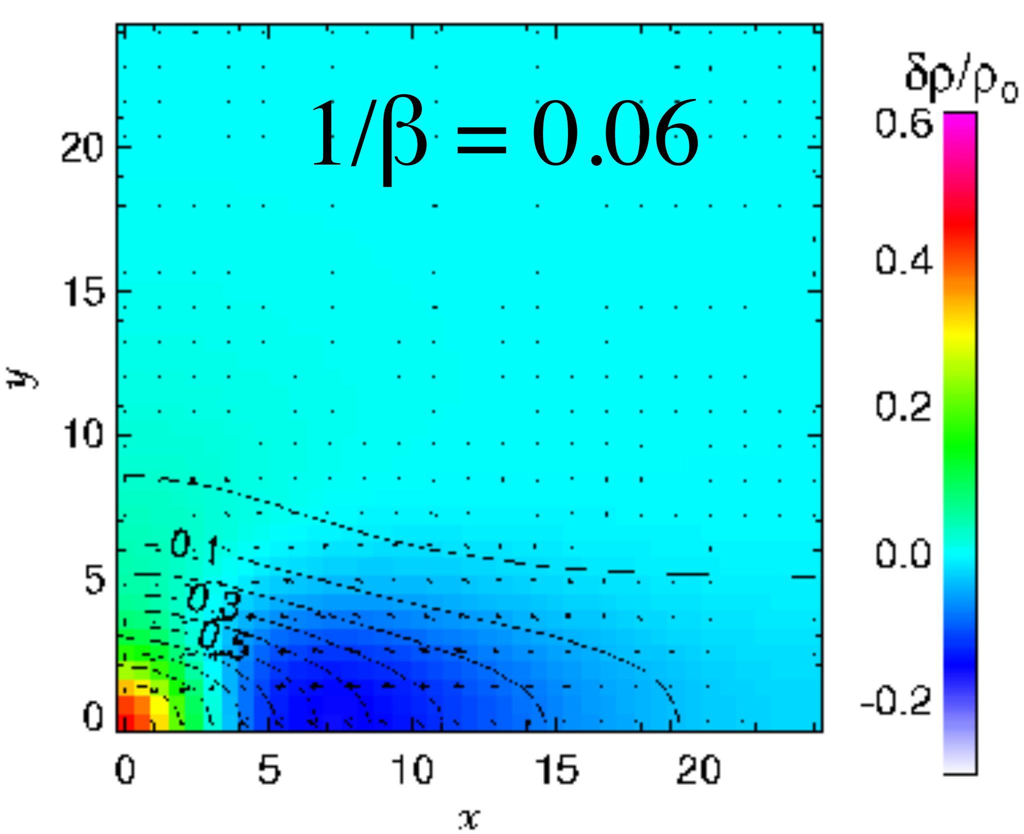
$\Delta x = \Delta y = 0.6\ H$,
$n_x = n_y = 40$

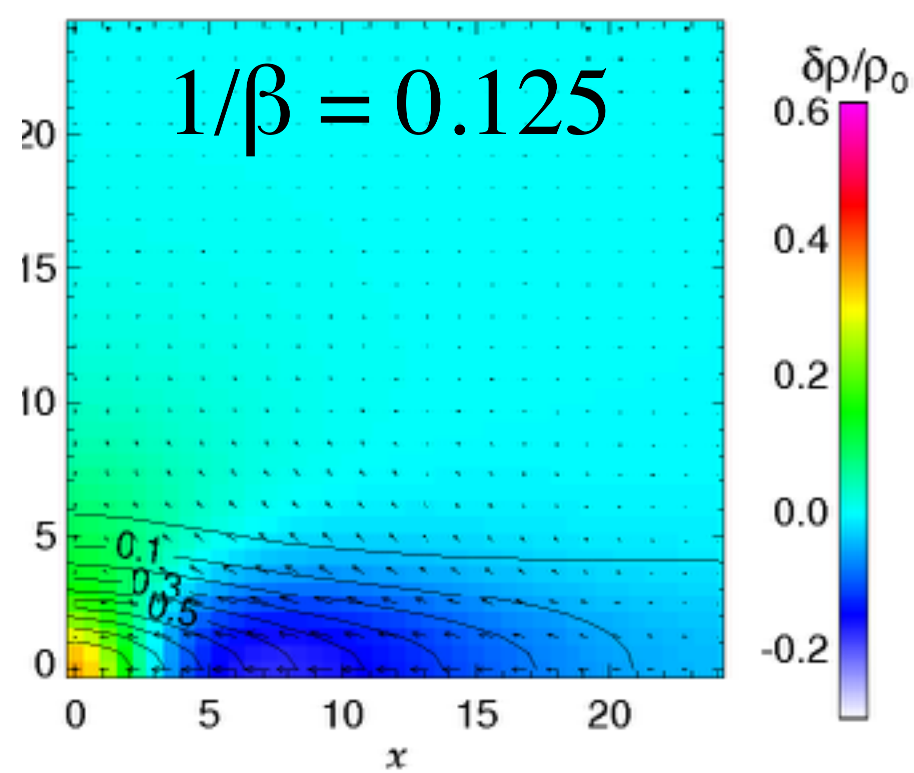


Eigen function $kH = 0.2$ normalization $\xi_z(0,0) = -H$





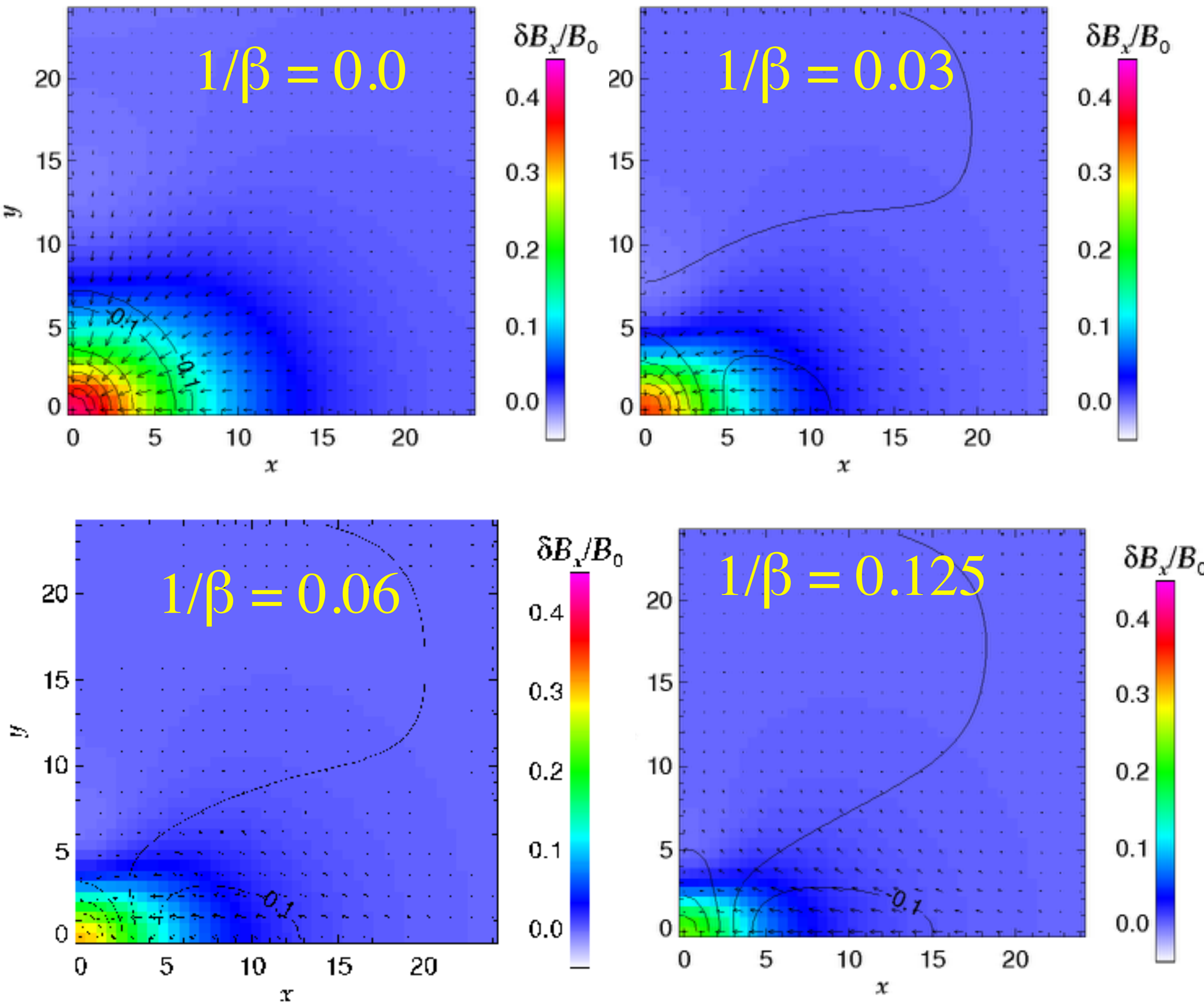




$$\left(\frac{\delta\varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial\xi_x}{\partial x} - \frac{\partial\xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial\xi_y}{\partial y} - k\xi_z$$

Change in B $kH = 0.2$ $\xi_z(0,0) = -H$

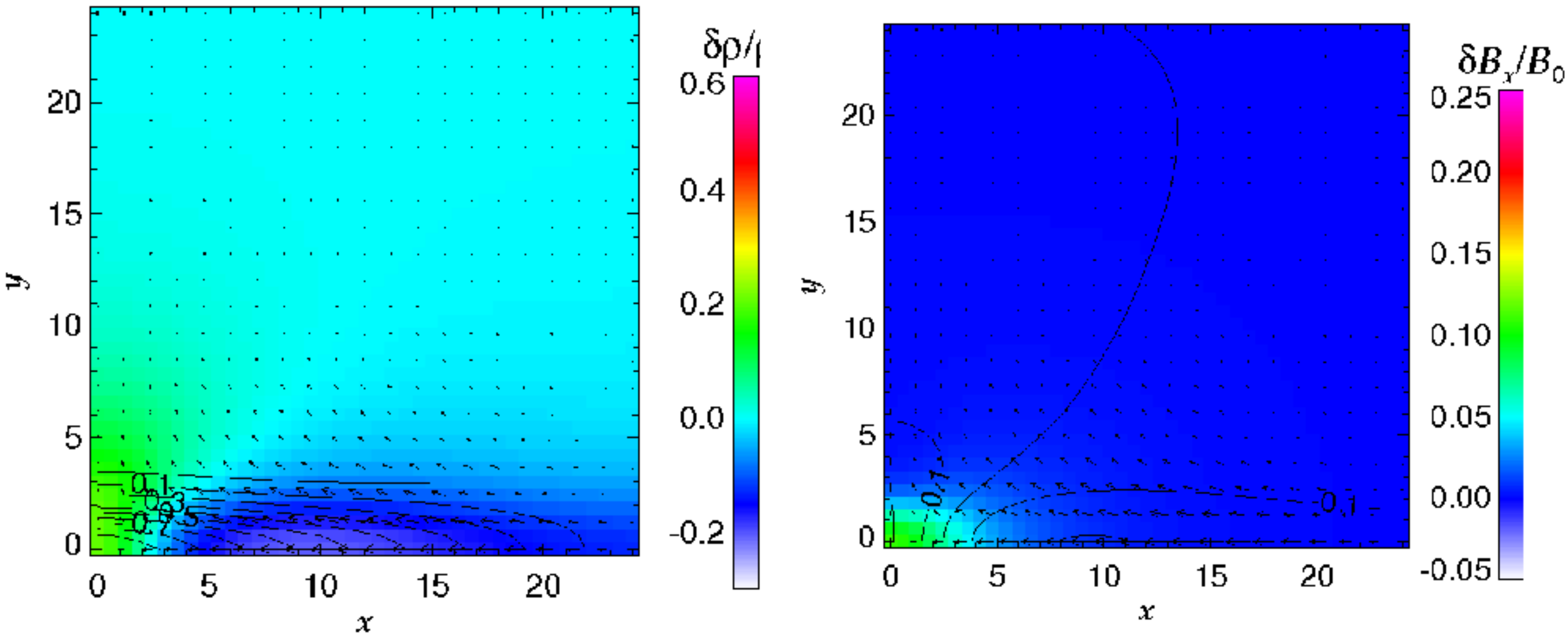


$$\left(\frac{\delta \varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_x}{\partial x} - \frac{\partial \xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_y}{\partial y} - k\xi_z$$

Further strong magnetic field ($kH = 0.2$)

$1/\beta = 0.375$



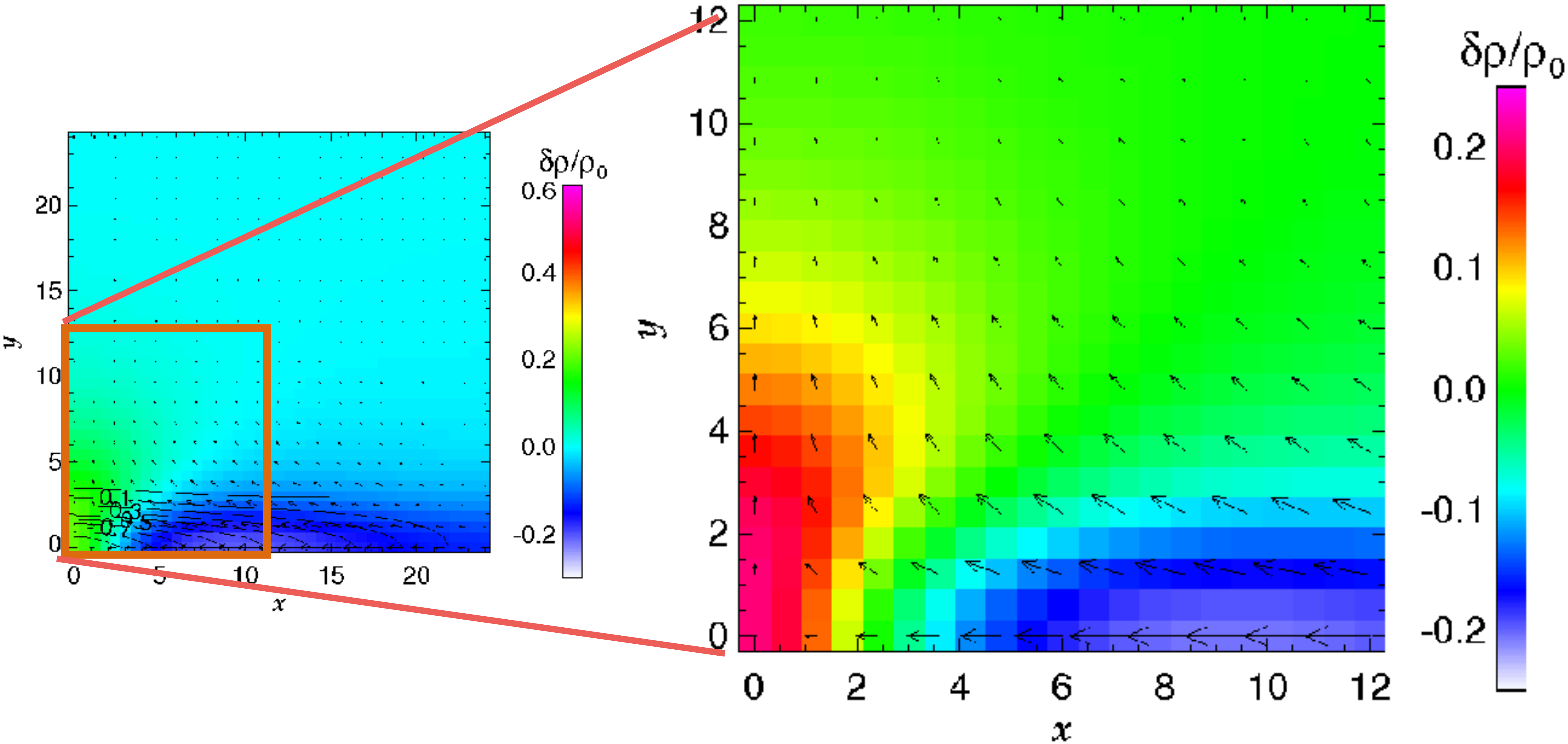
$\xi_z(0,0) = -H$

$$\left(\frac{\delta\varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial\xi_x}{\partial x} - \frac{\partial\xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial\xi_y}{\partial y} - k\xi_z$$

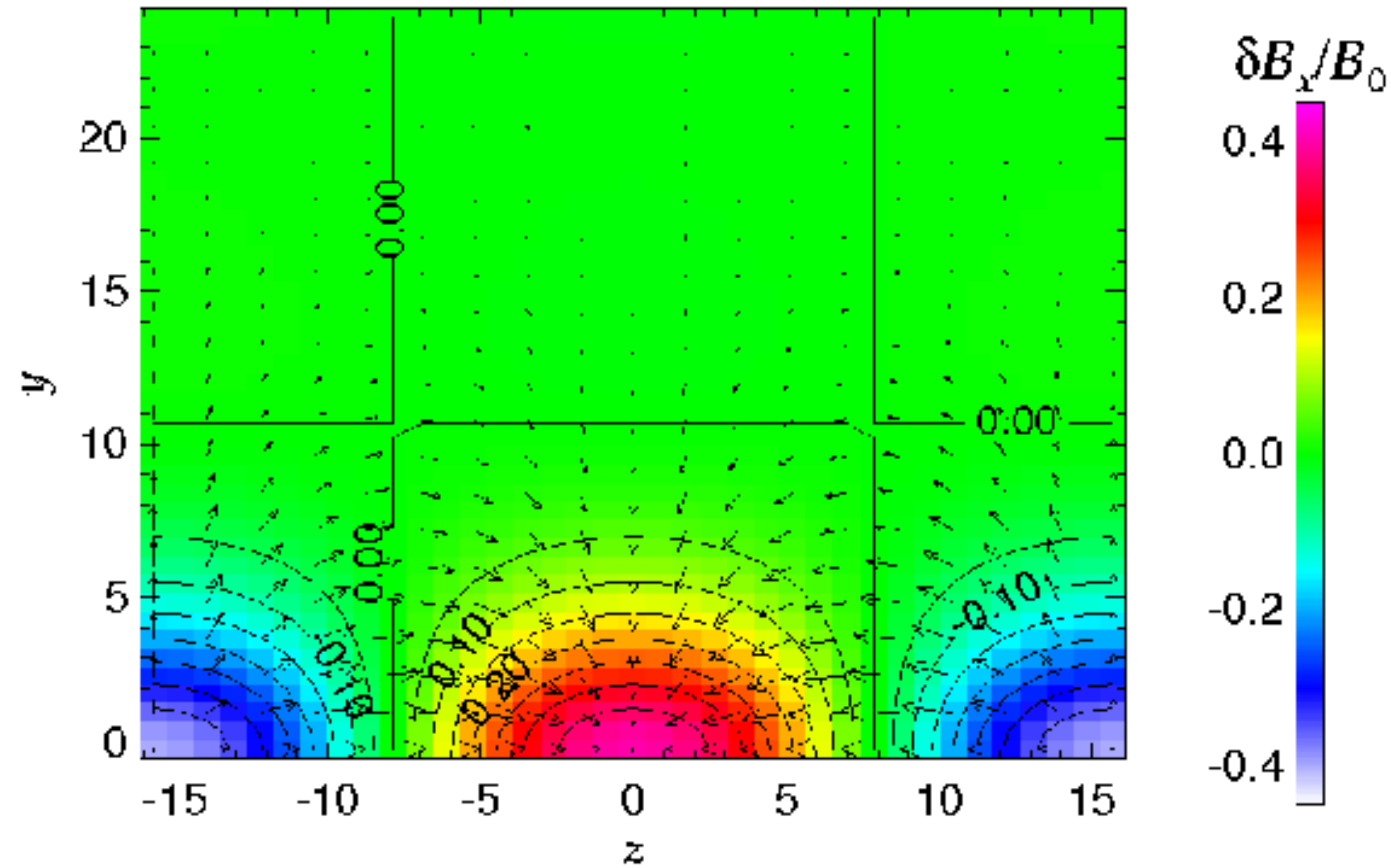
Enlargement

$1/\beta = 0.375, \quad kH = 0.2$

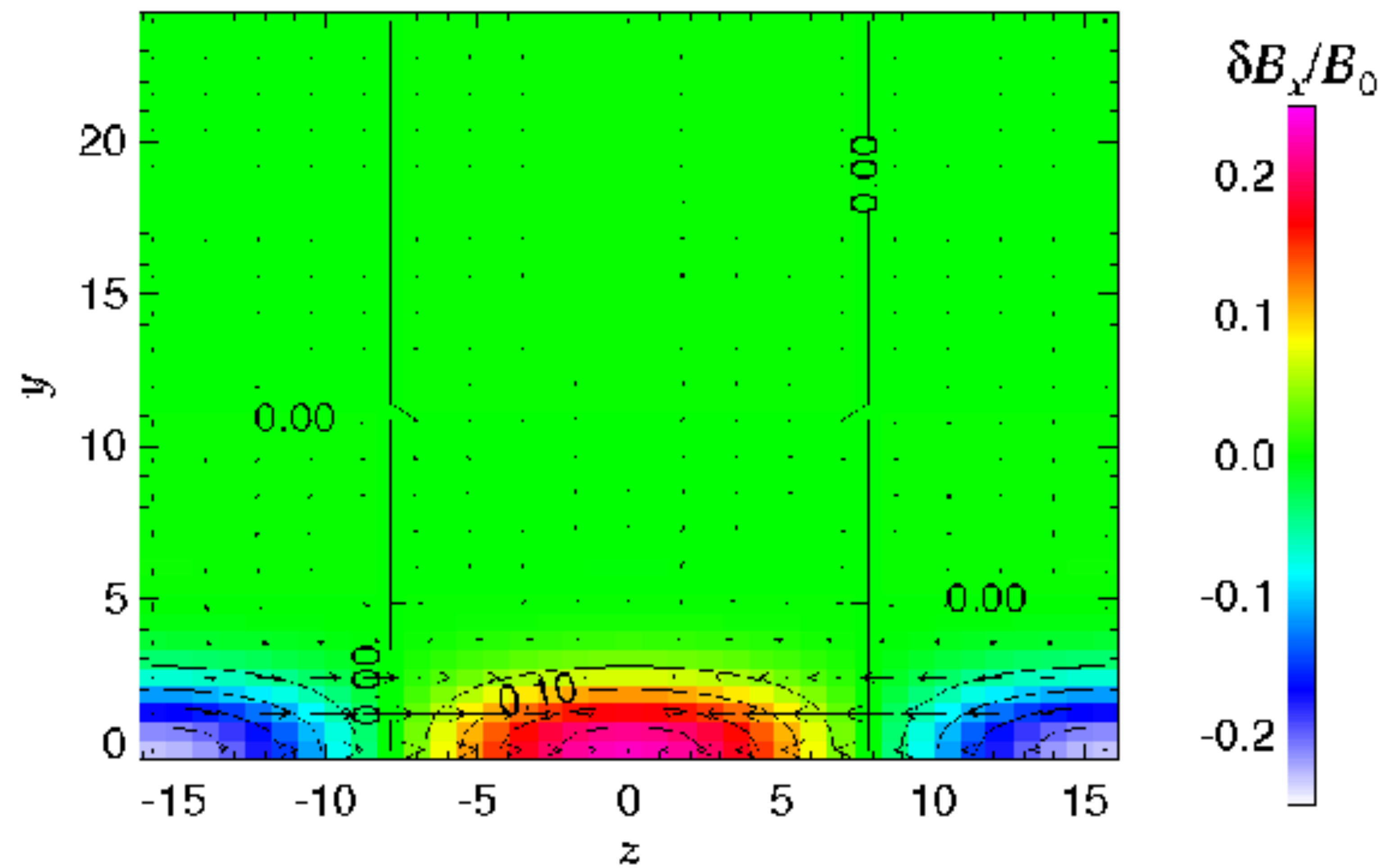


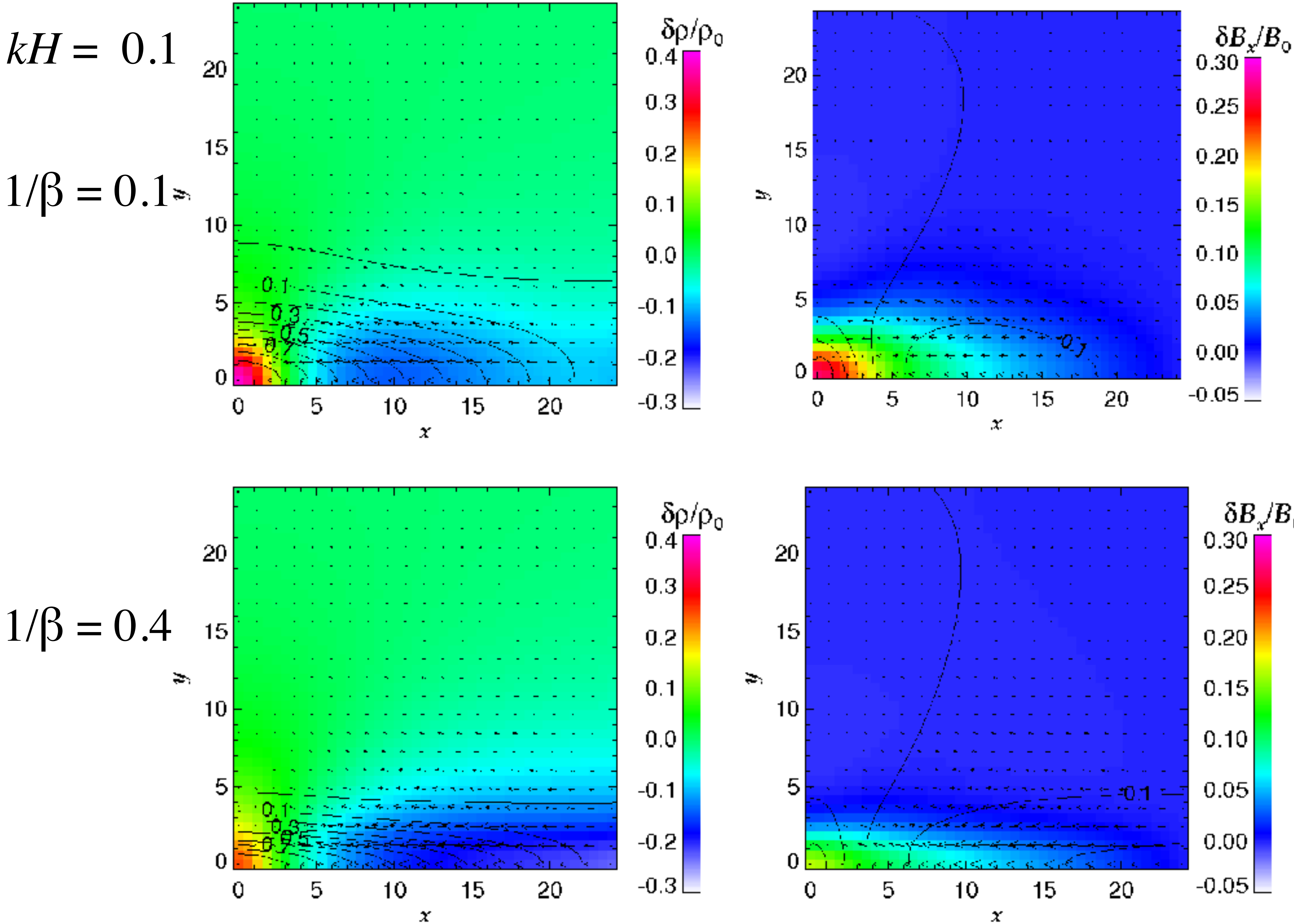
Flow in the *yz*-plane ($x = 0$)

$kH = 0.2,\ 1/\beta = 0.0$



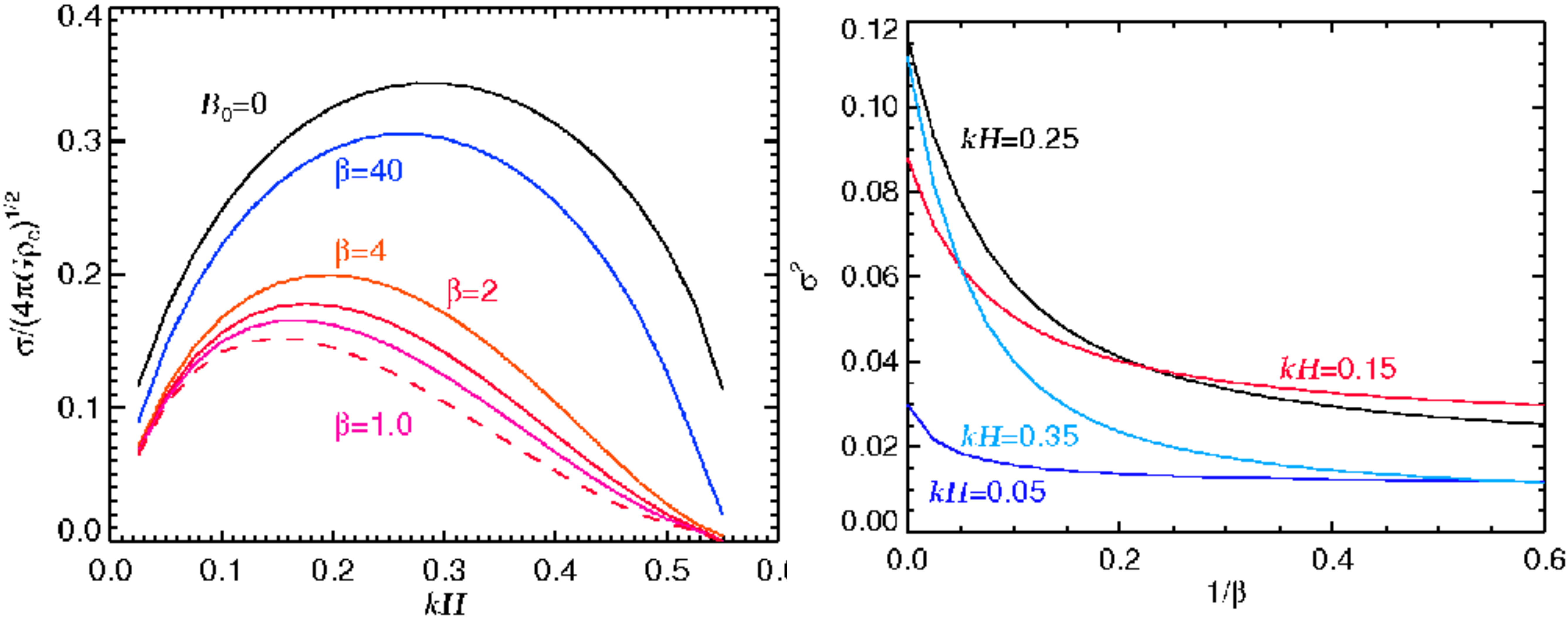
$kH = 0.2,\ 1/\beta = 0.125$





kH = 0.1
1/β = 0.1
1/β = 0.4
δρ/ρ0
δBx/B0
x
y
0.1
0.3
0.5
0.7

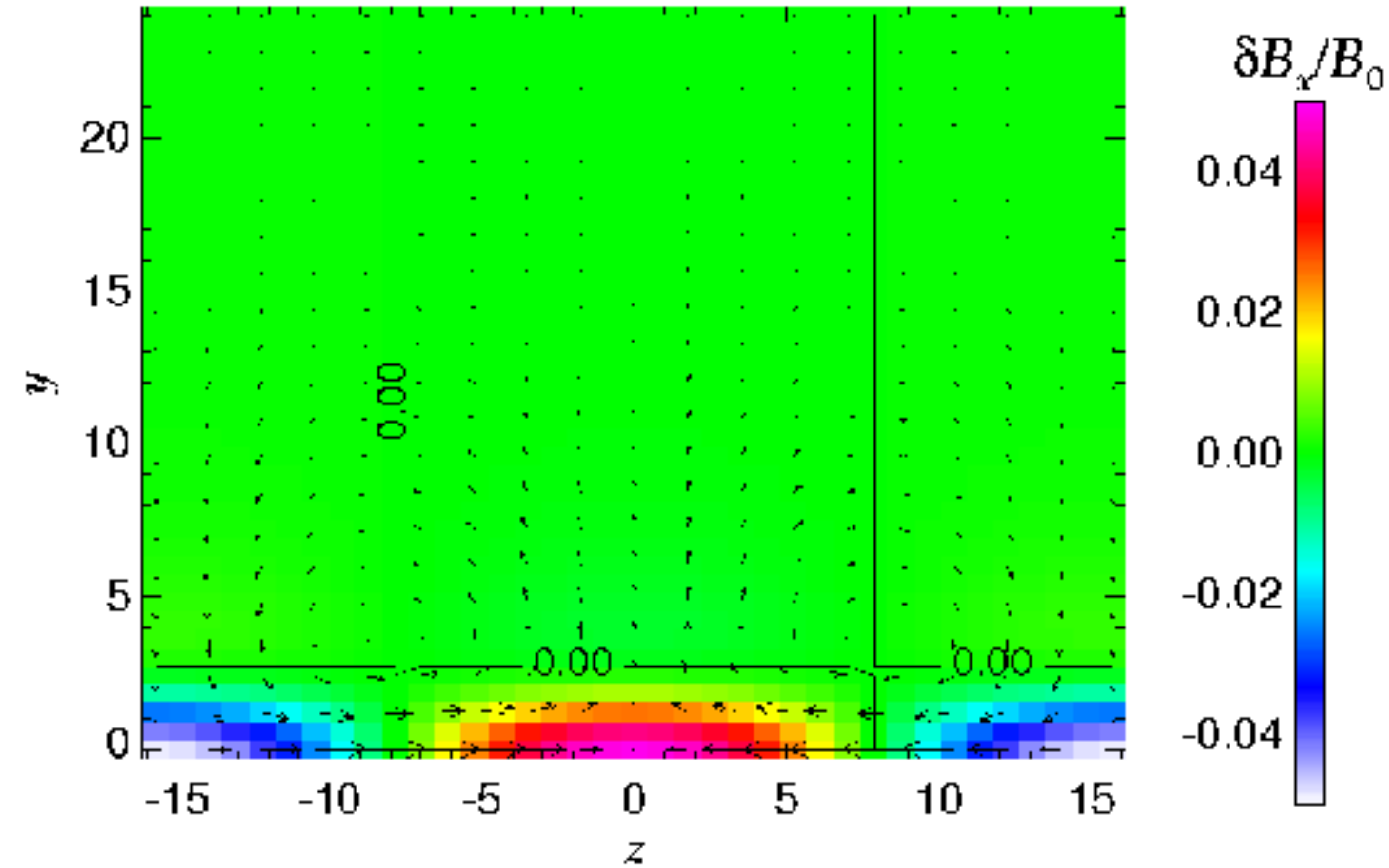
Free boundary

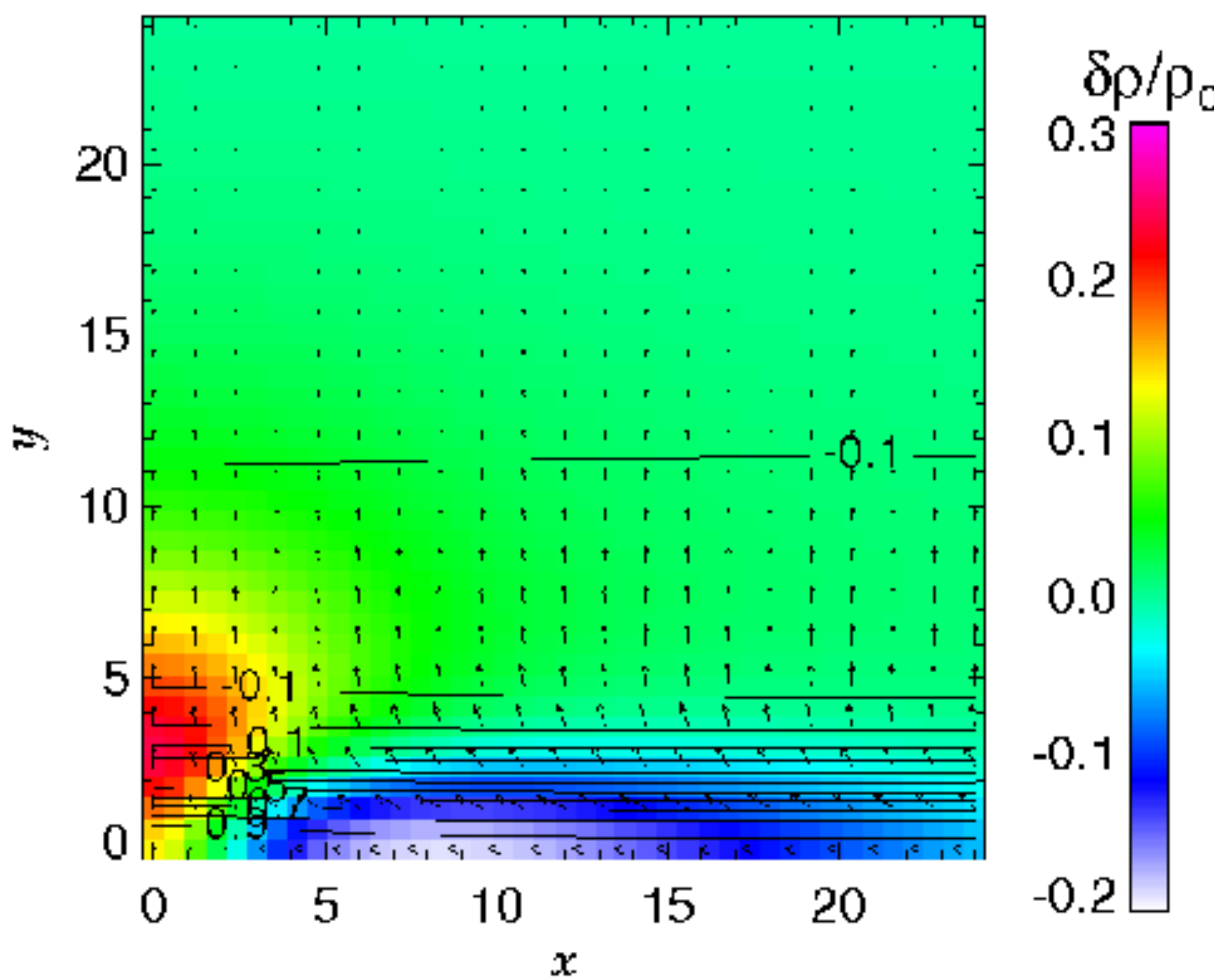


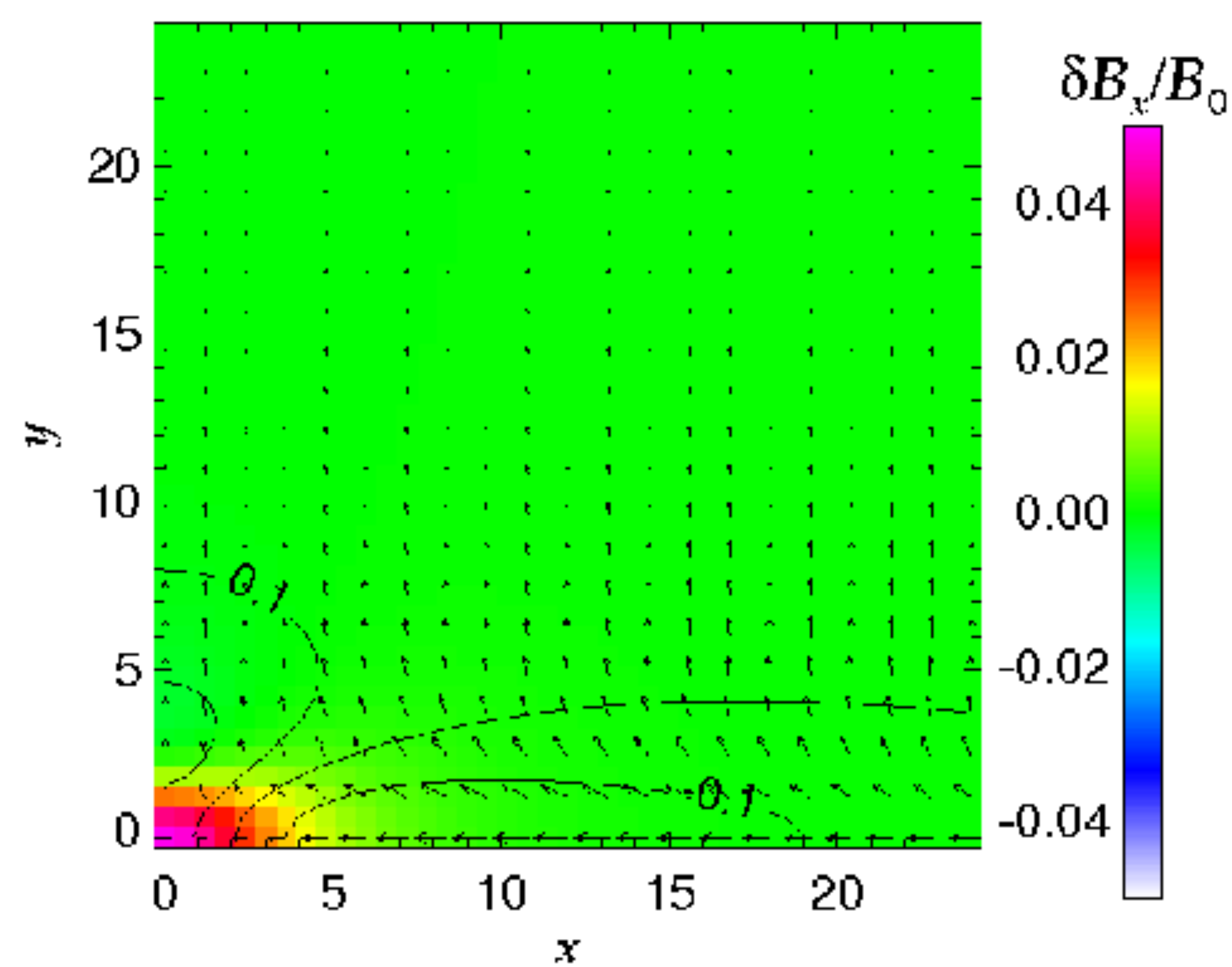
incompressive mode
(cf. Nagai+98)

Free boundary

$kH = 0.2, \beta = 0.5$

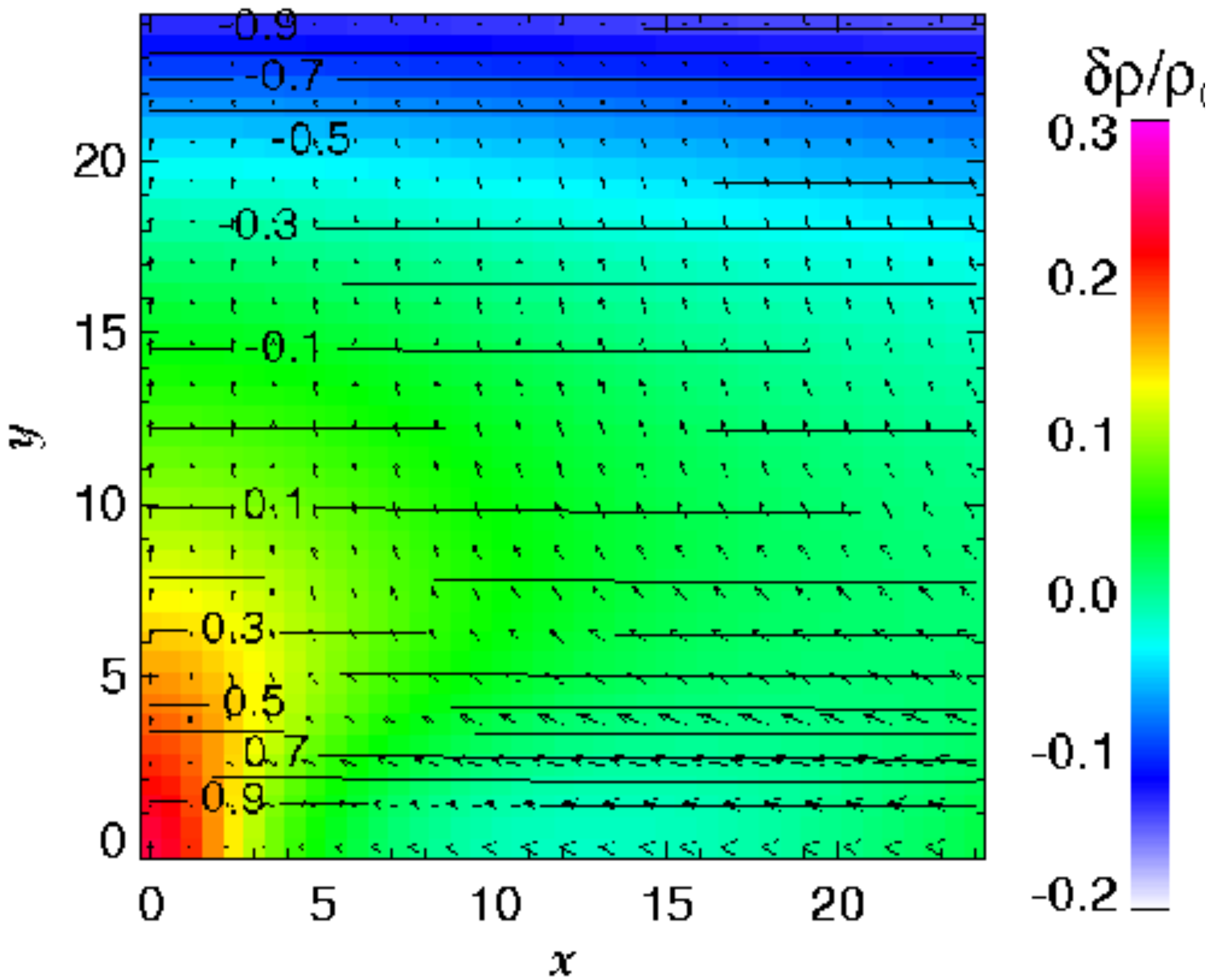


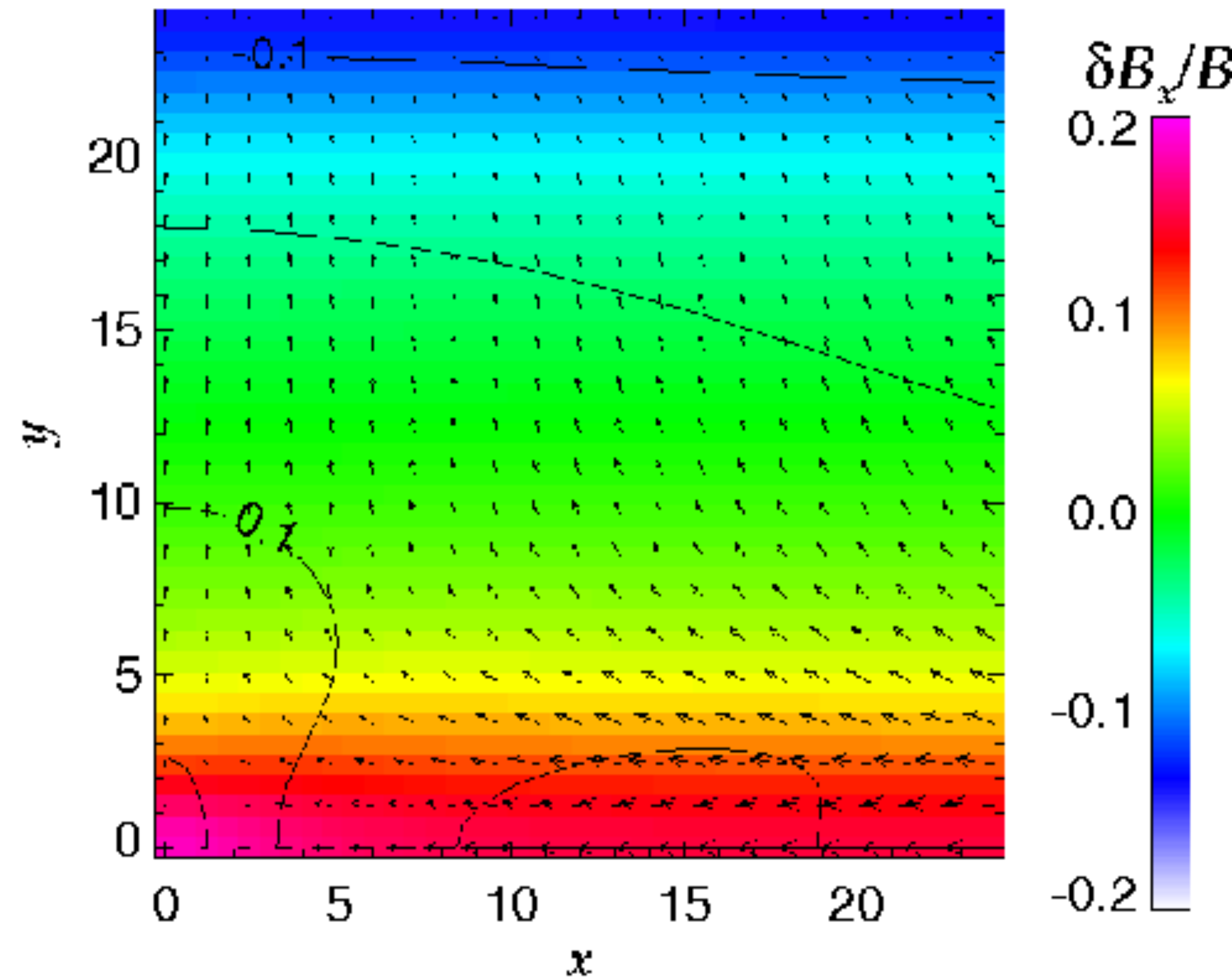




Free boundary

$kH = 0.05, \beta = 2$





Summary

- Vertical (uniform) magnetic field works against fragmentation.

- Compressible mode is suppressed by rather weak magnetic field.

- Incompressible mode survives even when B is extremely strong, if the magnetic field is not fixed on the boundary.

- Weak magnetic field affects flow in the low density region.