Numerical Computation of Electromagnetic Field for General Static and Axisymmetric Current Distribution and its Application to Poloidal Current Distribution in an ITER-like TOKAMAK

aka

Electromagnetic Field of Bagel

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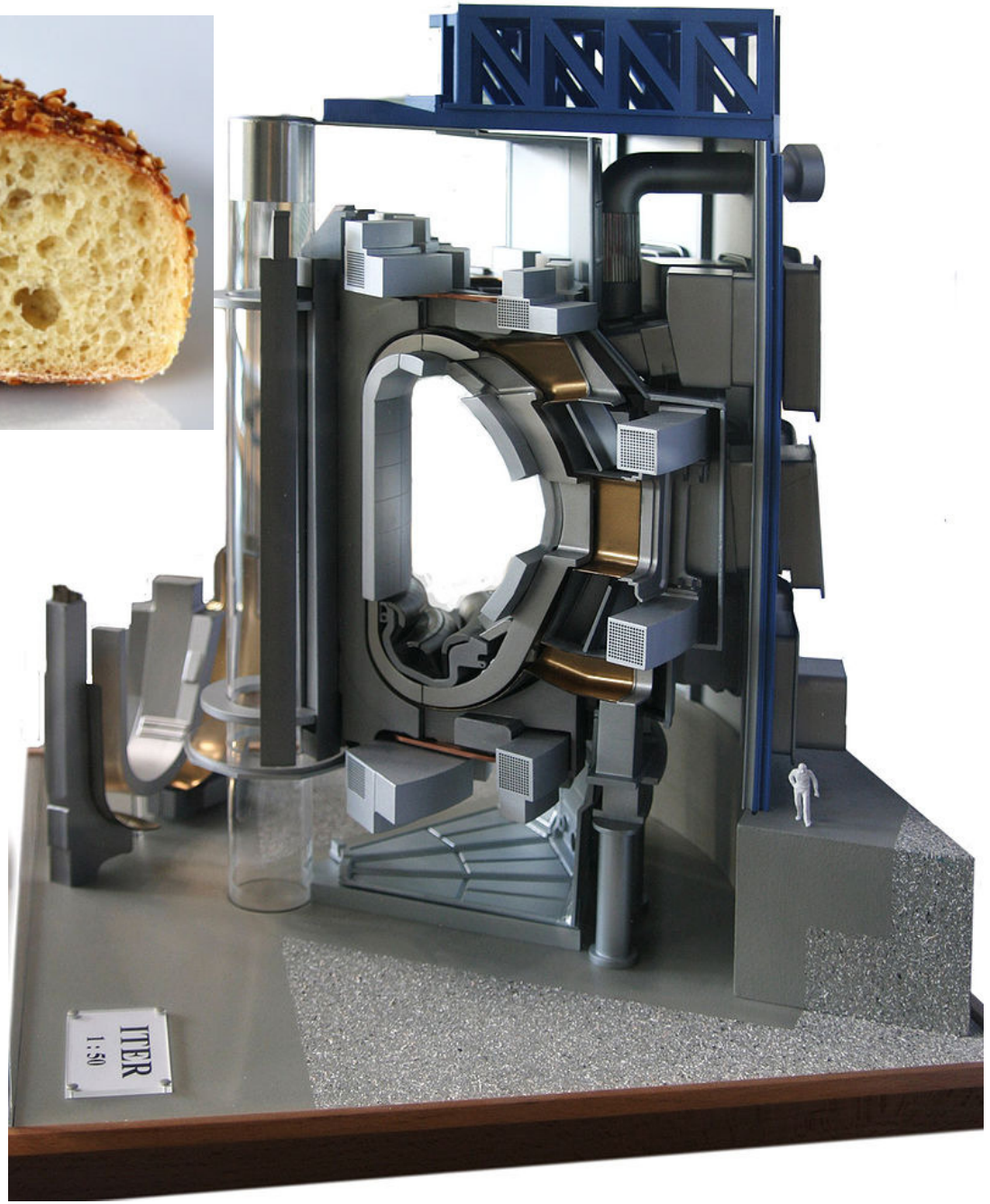
ResearchGate Fukushima Click


ITER
1:50

Coulomb, Biot, Savart

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\mathrm{d}^3\mathbf{x}'$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\mathrm{d}^3\mathbf{x}'$$

$$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \rho(\mathbf{x}')\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\mathrm{d}^3\mathbf{x}'$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \mathbf{J}(\mathbf{x}')\times\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\mathrm{d}^3\mathbf{x}'$$

Axisymmetric Electrostatic Potential

- Ring Potential Convolution (Jackson 1998)

$$\Phi(R,z)=\iint G(R',z';R,z)\,\rho(R',z')\,dR'\,dz'$$

- Green's Function by Complete Elliptic Integral

$$G(R',z';R,z)=\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{4R'K\left(m(R',z';R,z)\right)}{\sqrt{(R'+R)^2+(z'-z)^2}}$$

$$m(R',z';R,z)=\frac{4RR'}{(R'+R)^2+(z'-z)^2}$$

- Fast Computation of K(m) (Fukushima 2015)

Axisymmetric Magnetostatic Potential

- Ring Potential Convolution (Jackson 1998)

$$a(R,z) \equiv A(R,z)/R = \iint F(R',z';R,z)\,J(R',z')\,dR'\,dz'$$

- Green's Function by Complete Elliptic Integral

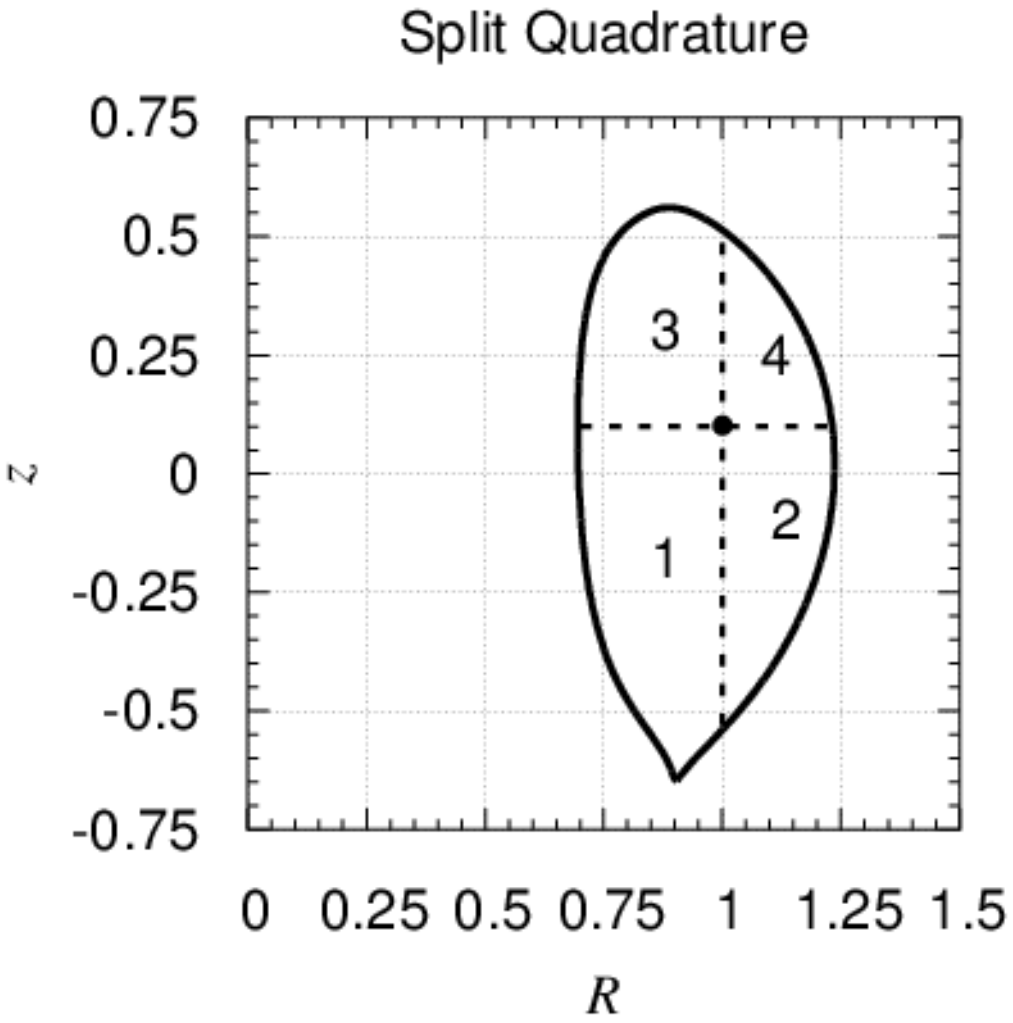
$$F(R',z';R,z) = \left(\frac{\mu_0}{4\pi}\right)\frac{16(R')^2\,S\left(m(R',z';R,z)\right)}{\left[\sqrt{(R'+R)^2+(z'-z)^2}\right]^3}$$

$$S(m) \equiv \frac{(2-m)K(m)-E(m)}{m^2}$$

- Fast Computation of S(m) (Fukushima 2016b)

Split Quadrature

- Avoid Singularity
- Split into 4 Parts
  at R=R′ & z=z′
- Using Double Exponential Rules (Takahashi & Mori, 1976; Ooura 2006)



Axisymmetric Field Computation

- Numerical Differentiation of Numerically Integrated Potentials by Central Diff. Formula

$$E_R(R,z) \approx \frac{\Phi(R+\Delta R,z)-\Phi(R-\Delta R,z)}{2\Delta R}$$

$$B_R(R,z) \approx -R\left[\frac{a(R,z+\Delta z)-a(R,z-\Delta z)}{2\Delta z}\right]$$

$$E_z(R,z) \approx \frac{\Phi(R,z+\Delta z)-\Phi(R,z-\Delta z)}{2\Delta z}$$

$$B_z(R,z) \approx 2a(R,z)+R\left[\frac{a(R+\Delta R,z)-a(R-\Delta R,z)}{2\Delta R}\right]$$

- More Precise Derivatives -> Ridder's Method
  - Ridder (1982)
  - Example: Gravitational Field (Fukushima 2016a)

Comparison with Exact Solutions

- Newly Obtained Exact Solutions

$$\rho_G(R,z)=\rho_0 r(\xi,\zeta)e(\xi,\zeta) \qquad J_G(R,z)=J_0 q(\xi,\zeta)e(\xi,\zeta)$$

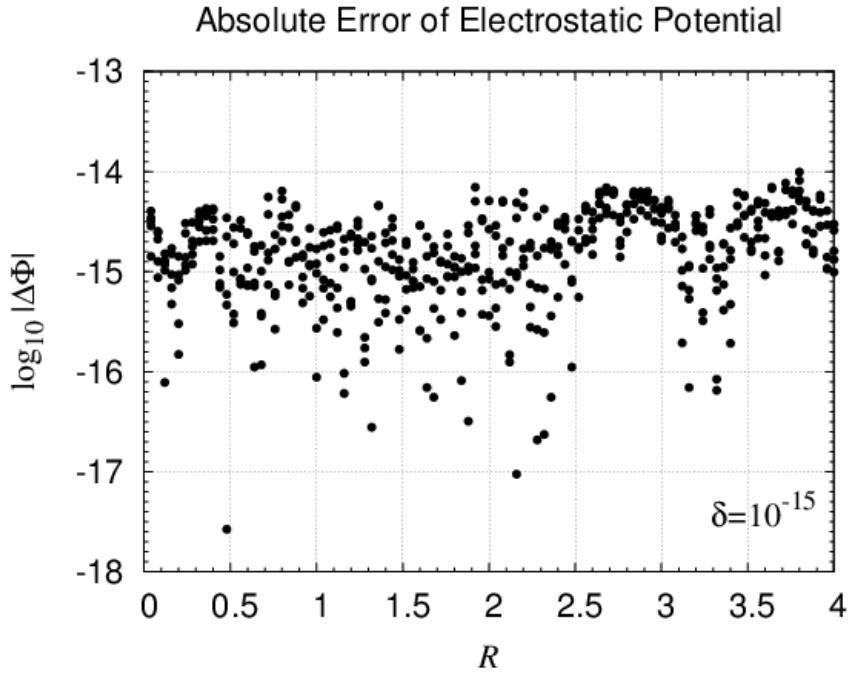
$$\Phi_G(R,z)=(\rho_0/\varepsilon_0)\xi^3 e(\xi,\zeta) \qquad A_G(R,z)=\varepsilon_0 J_0 \xi^3 e(\xi,\zeta)$$

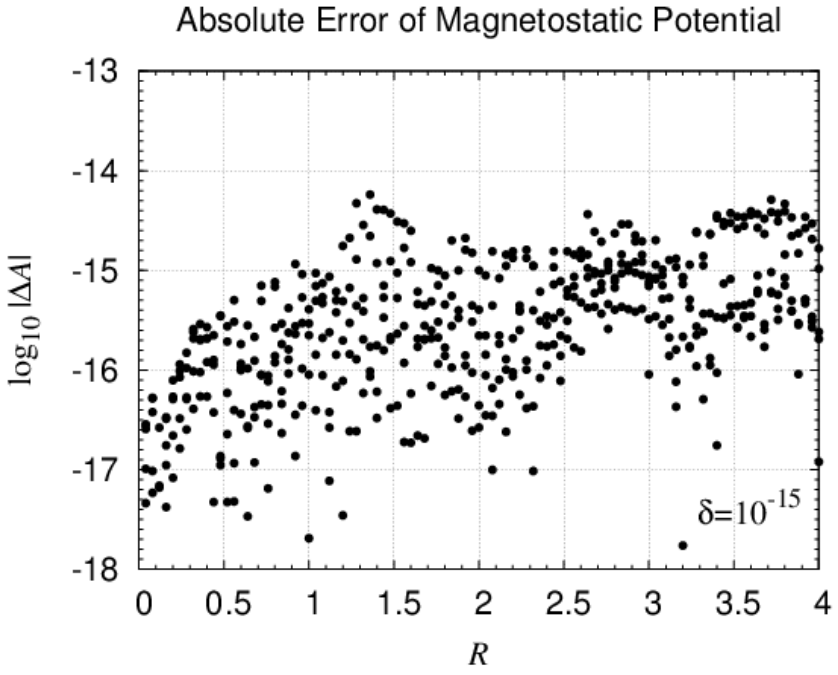
- Auxiliary Functions/Quantities

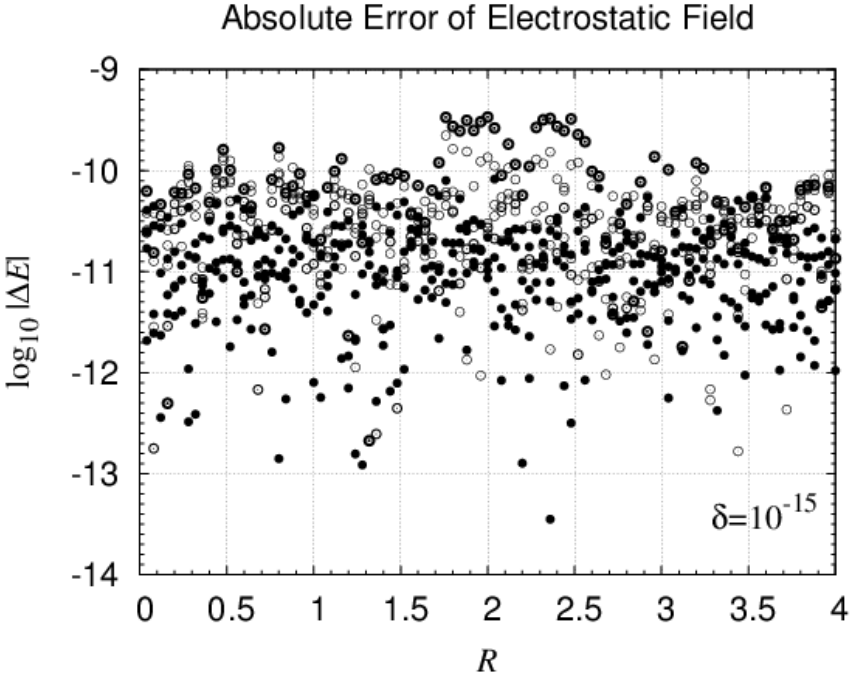
$$e(\xi,\zeta)=\exp\left[-(\xi-\xi_0)^2-\zeta^2\right] \qquad \xi=R/H_R,\xi_0=R_0/H_R,\zeta=z/H_z$$

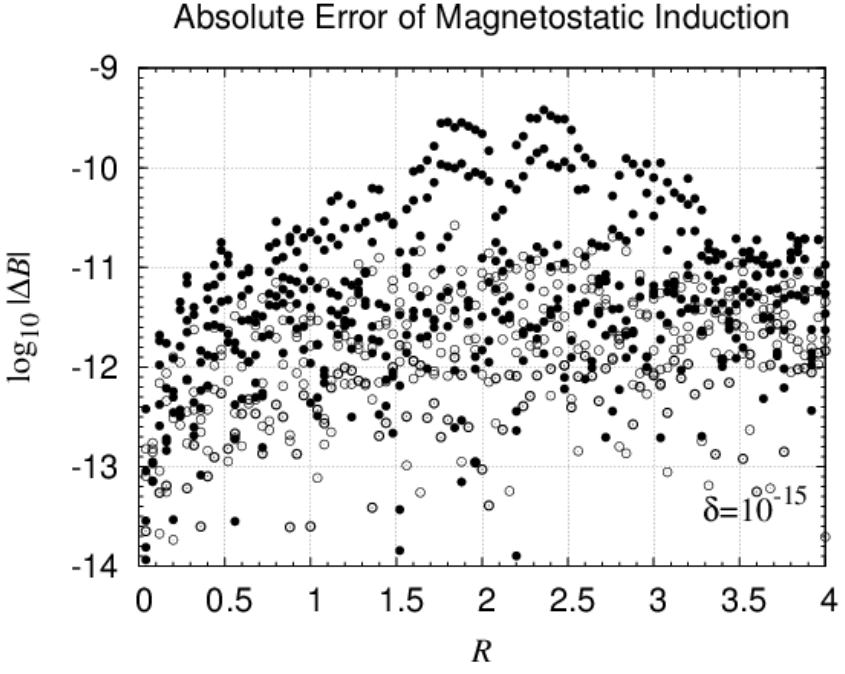
$$q(\xi,\zeta)=-8\xi-14\xi_0\xi^2+q_3(\zeta)\xi^3+8\xi_0\xi^4-4\xi^5$$

$$r(\xi,\zeta)=q(\xi,\zeta)-\xi \qquad q_3(\zeta)=16-4\xi_0^2+2(H_R/H_z)^2(1-2\zeta^2)$$


Absolute Error of Electrostatic Potential
-13
-14
-15
-16
-17
-18
0
0.5
1
1.5
2
2.5
3
3.5
4
R
log10 |ΔΦ|
δ=10-15

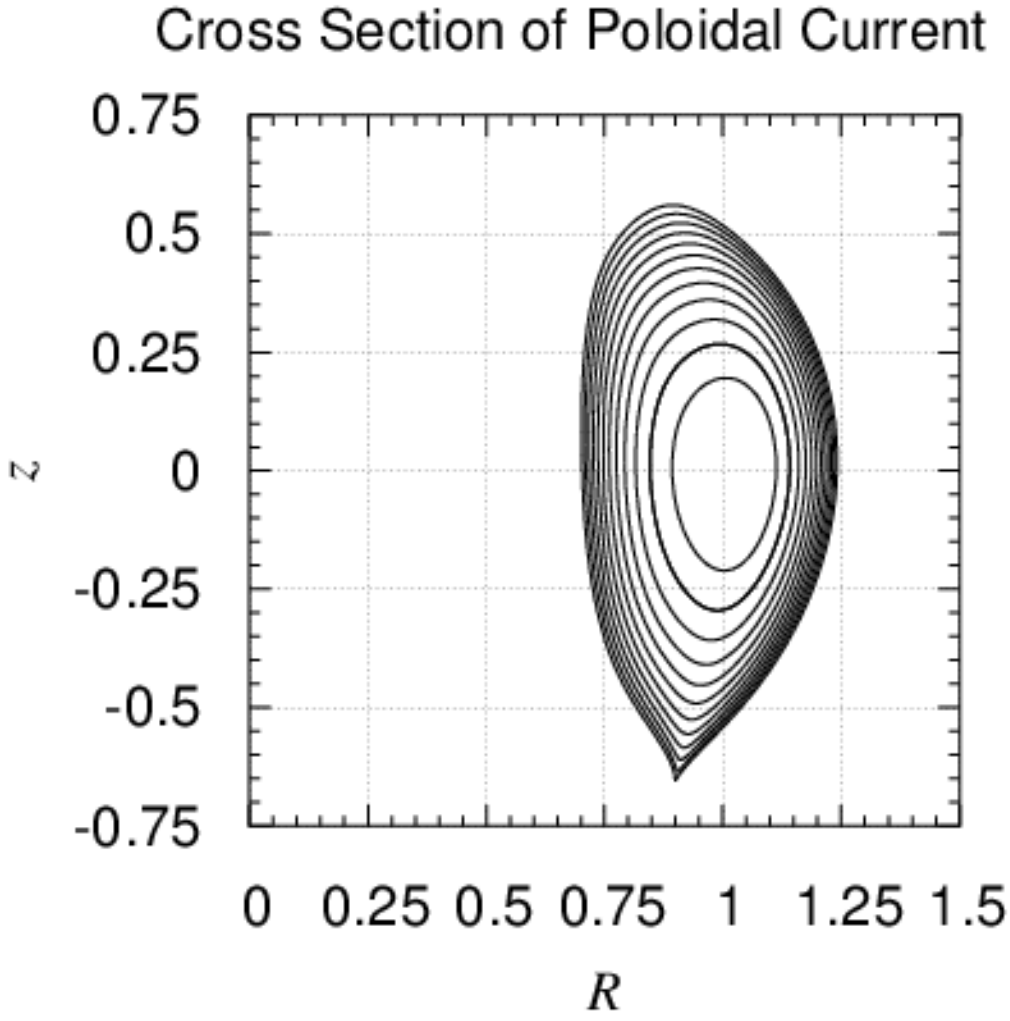

Absolute Error of Magnetostatic Potential
-13
-14
-15
-16
-17
-18
0
0.5
1
1.5
2
2.5
3
3.5
4
R
log10 |ΔA|
δ=10-15


Absolute Error of Electrostatic Field
-9
-10
-11
-12
-13
-14
0
0.5
1
1.5
2
2.5
3
3.5
4
R
log10 |ΔE|
δ=10-15

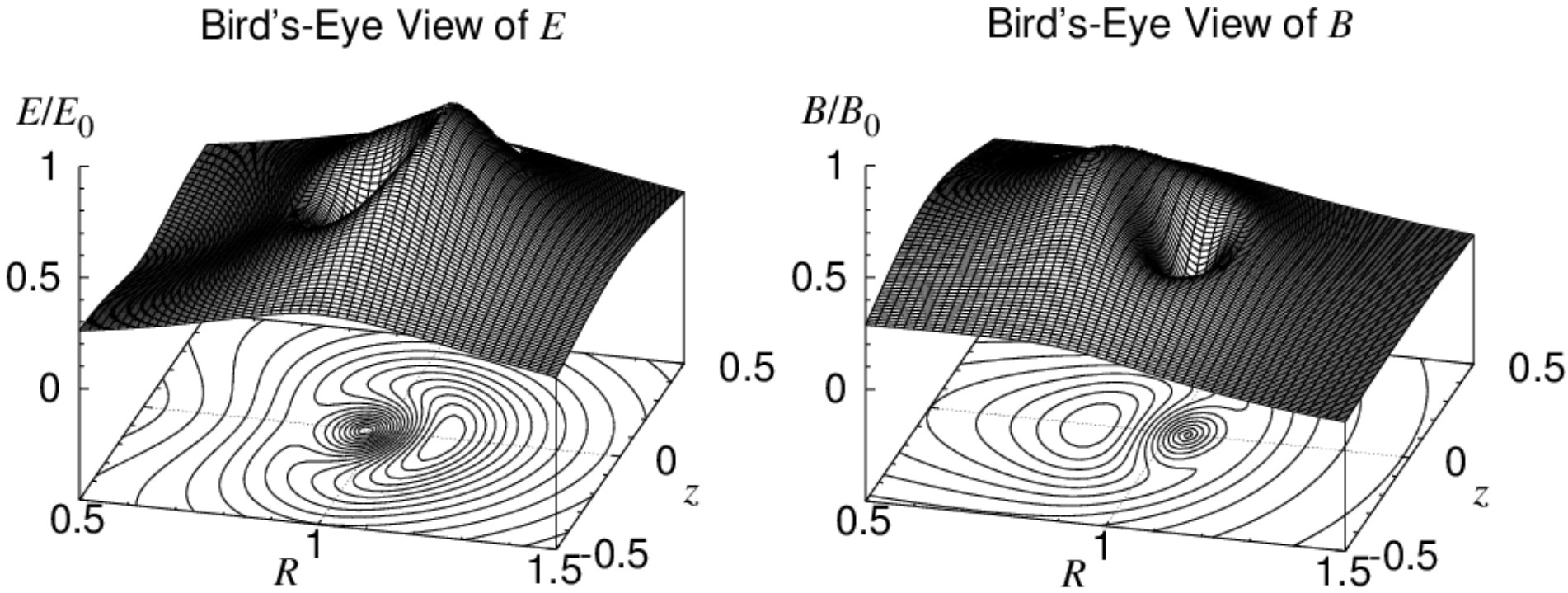

Absolute Error of Magnetostatic Induction
-9
-10
-11
-12
-13
-14
0
0.5
1
1.5
2
2.5
3
3.5
4
R
log10 |ΔB|
δ=10-15

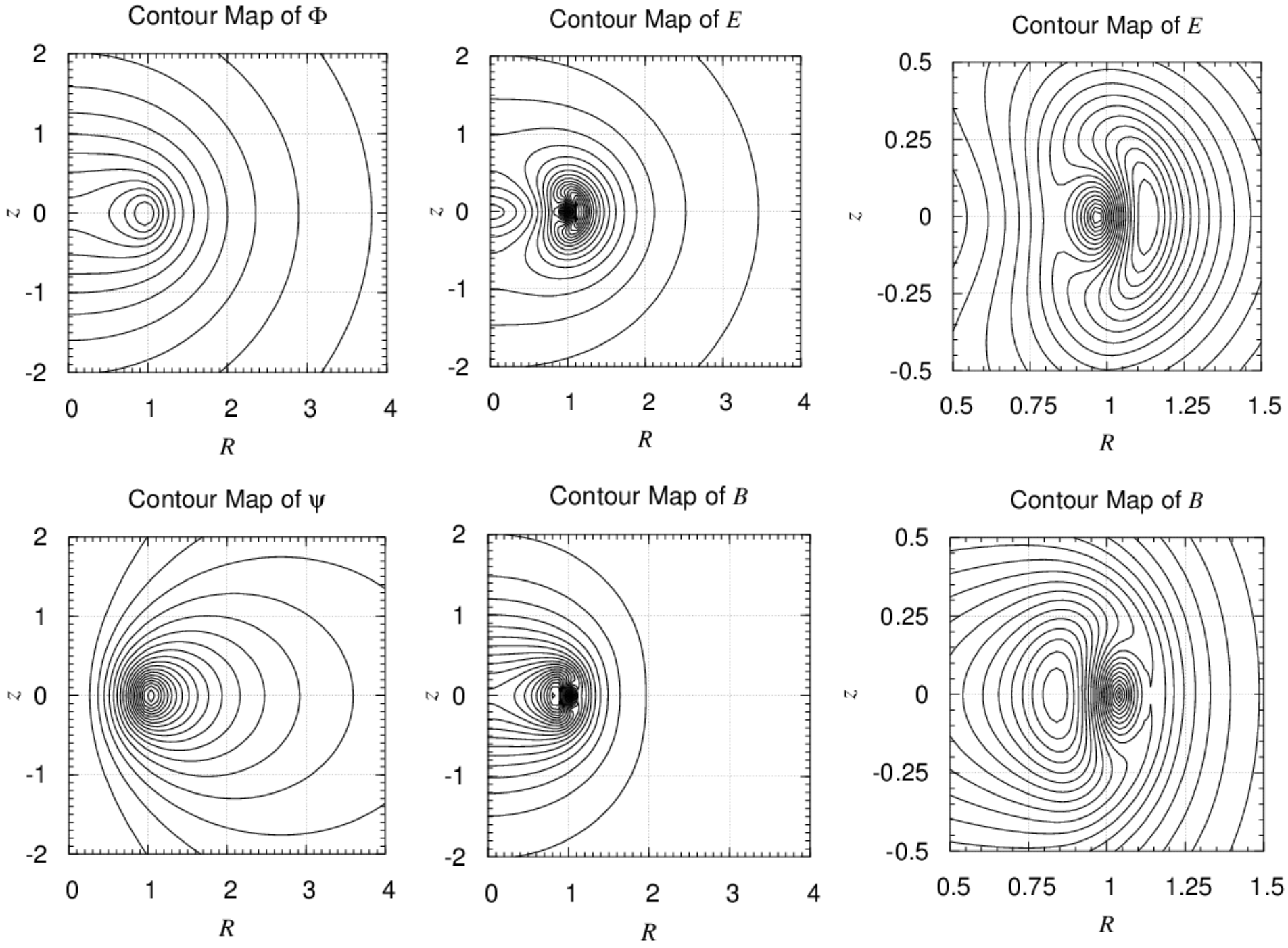
Sample Poloidal Current Distribution

- Based on Evangelias & Throumoulopoulos (2016)



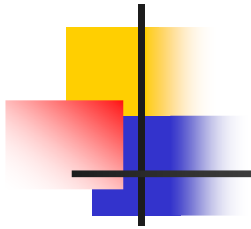
EM Field of Sample Poloidal Current Distr.




Contour Map of Φ
Contour Map of E
Contour Map of E
Contour Map of ψ
Contour Map of B
Contour Map of B
z
R
2
1
0
-1
-2
0
1
2
3
4
0.5
0.25
0
-0.25
-0.5
0.5
0.75
1
1.25
1.5

Conclusion

- New Computation of Axisymmetric Electrostatic/Magnetostatic Fields
  - Potential Integration by Split Quadrature
  - Differentiation by Central Diff. Formula
  - 9-14 digit Accuracy
- Sample Computation of EM Field by Poloidal Current Distribution
  - X-point Effect is Negligibly Small



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