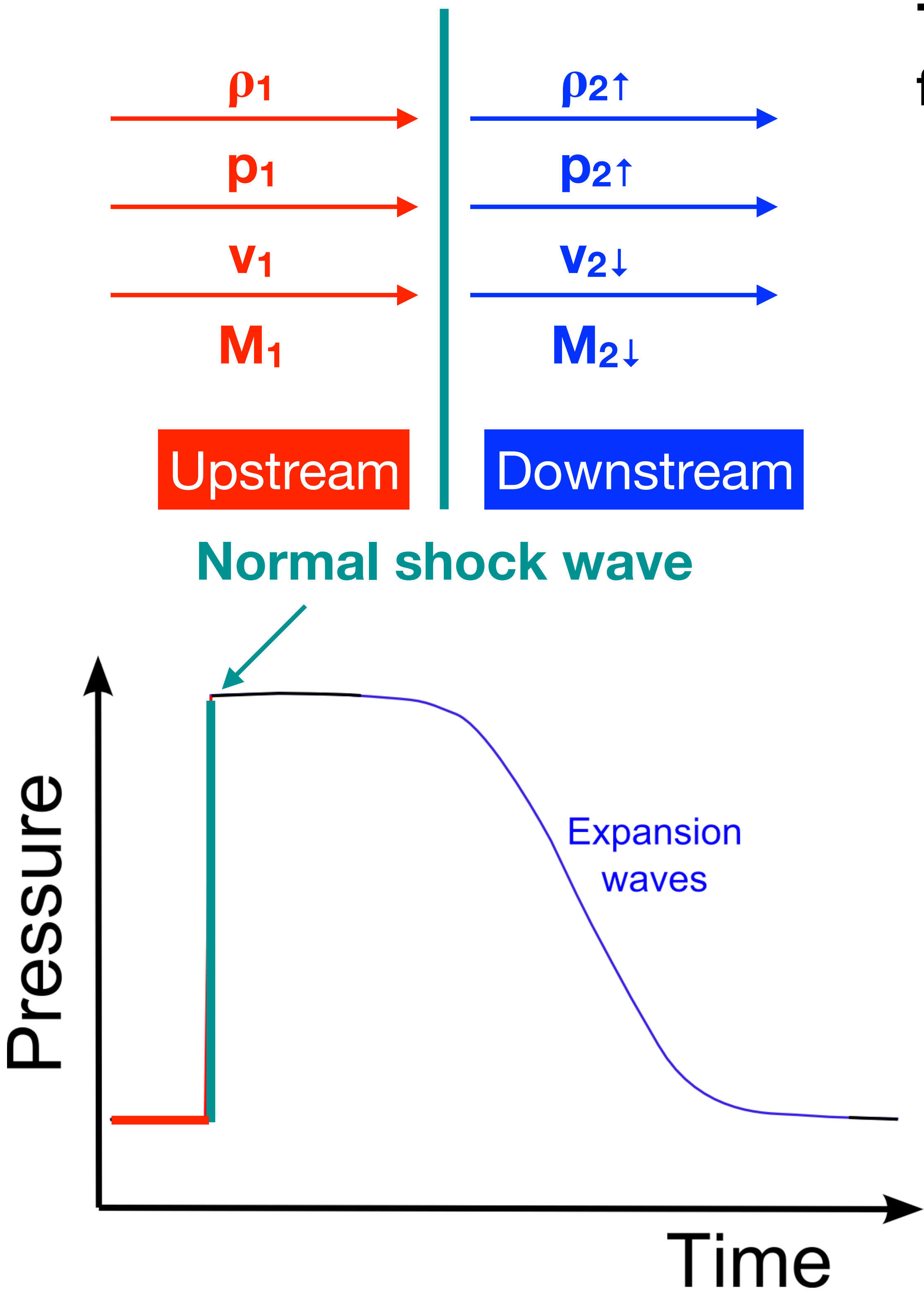
**Arnab Rai Choudhuri**

# Section 6.5-6.8

2018/11/30

Yuzhu, Cui

# 6.5 The structure of shock waves




**The mass flux, the momentum flux and the energy flux is conserved under the steady conditions:**

$$\rho_1 v_1 = \rho_2 v_2 \tag{6.33}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{6.34}$$

$$\frac{1}{2}v_1^2 + \frac{\gamma p_1}{(\gamma - 1)\rho_1} = \frac{1}{2}v_2^2 + \frac{\gamma p_2}{(\gamma - 1)\rho_2} \tag{6.35}$$

Mach number: $\mathscr{M} = \frac{v_1}{\sqrt{\gamma p_1/\rho_1}} = \frac{v_1}{c_{s,1}}$

$\mathscr{M} > 1$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\mathscr{M}^2}{2 + (\gamma - 1)\mathscr{M}^2} = \frac{\gamma + 1}{\frac{2}{\mathscr{M}^2} + (\gamma - 1)} \tag{6.36}$$

$$\frac{\rho_2}{\rho_1} \rightarrow \frac{\gamma + 1}{\gamma - 1} \quad (\mathscr{M} \rightarrow \infty) \tag{6.39}$$

$$\frac{p_2}{p_1} = \frac{2\gamma\mathscr{M}^2 - (\gamma - 1)}{\gamma + 1} \tag{6.38}$$

# 6.5 The structure of shock waves

National Aeronautics and Space Administration

## Normal Shock Wave

Zone 0 Upstream — $\gamma$ – specific heat ratio — Zone 1 Downstream

| Zone 0 Upstream | | Zone 1 Downstream |
|---|---|---|
| p – static pressure | Shock Wave | p – static pressure |
| $p_t$ – total pressure | | $p_t$ – total pressure |
| T – static temperature | Flow → | T – static temperature |
| $T_t$ – total temperature | | $T_t$ – total temperature |
| $\rho$ – density | | $\rho$ – density |
| M – Mach number | | $M_1$ – Mach number |

$$\frac{p_1}{p_0} = \frac{2\gamma M^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{p_{t_1}}{p_{t_0}} = \left[\frac{(\gamma+1) M^2}{(\gamma-1) M^2 + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{(\gamma+1)}{2\gamma M^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{T_1}{T_0} = \frac{[2\gamma M^2 - (\gamma-1)]\,[(\gamma-1) M^2 + 2]}{(\gamma+1)^2 M^2}$$

$$\frac{T_{t_1}}{T_{t_0}} = 1$$

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1) M^2}{(\gamma-1) M^2 + 2}$$

$$M_1^2 = \frac{(\gamma-1) M^2 + 2}{2\gamma M^2 - (\gamma-1)}$$

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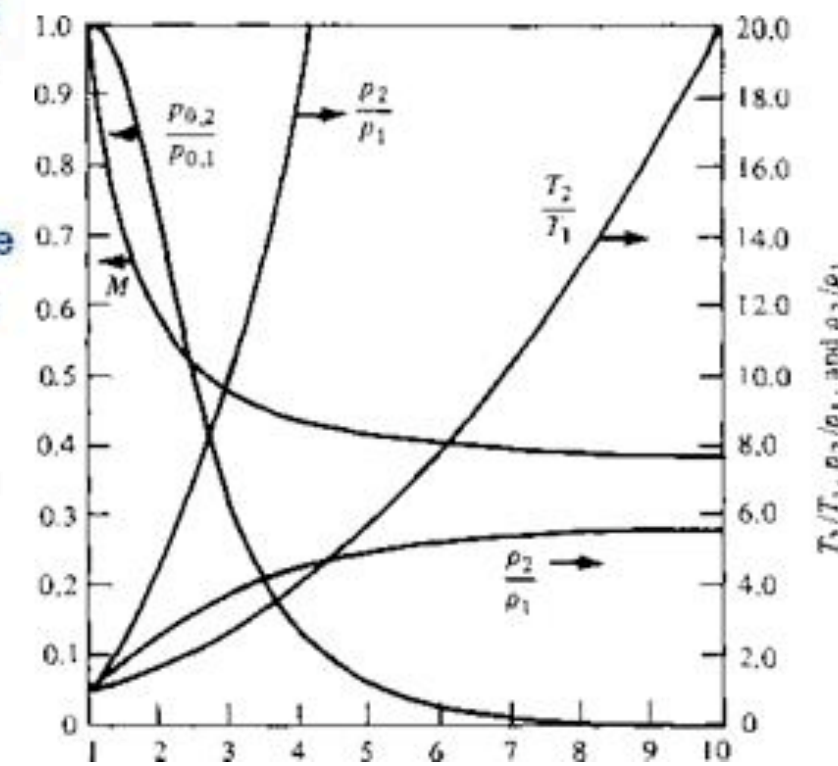
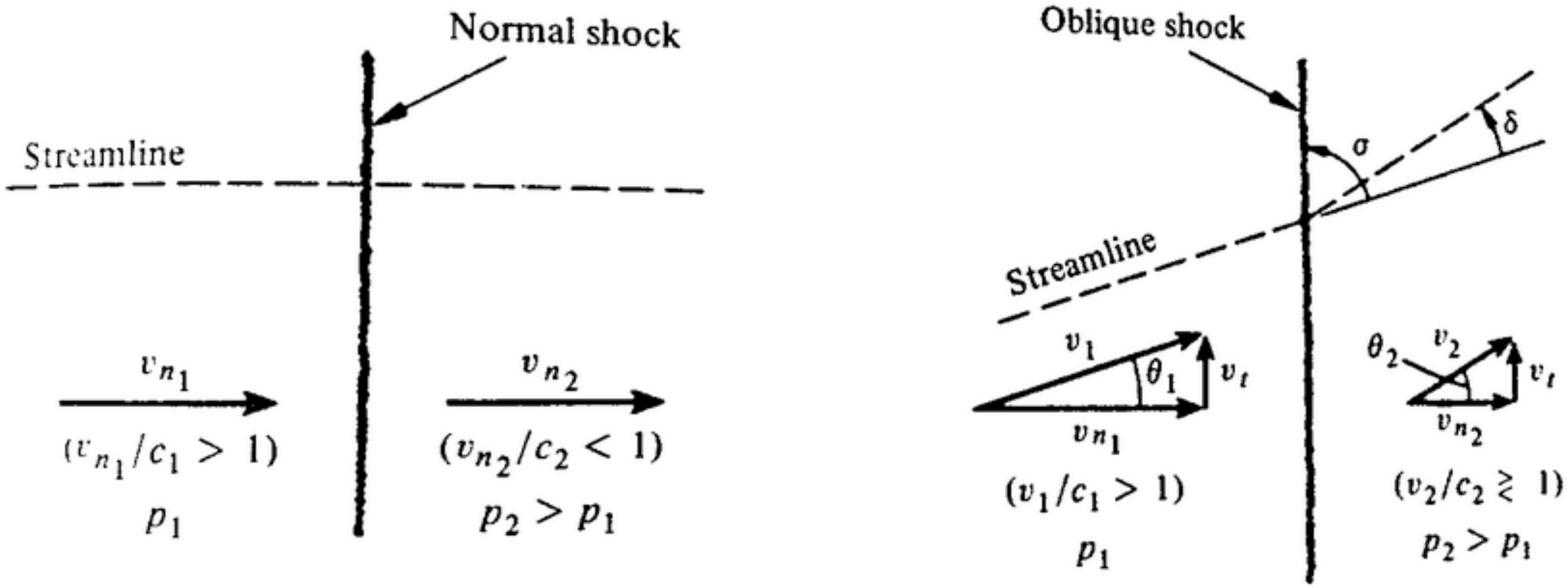



Time

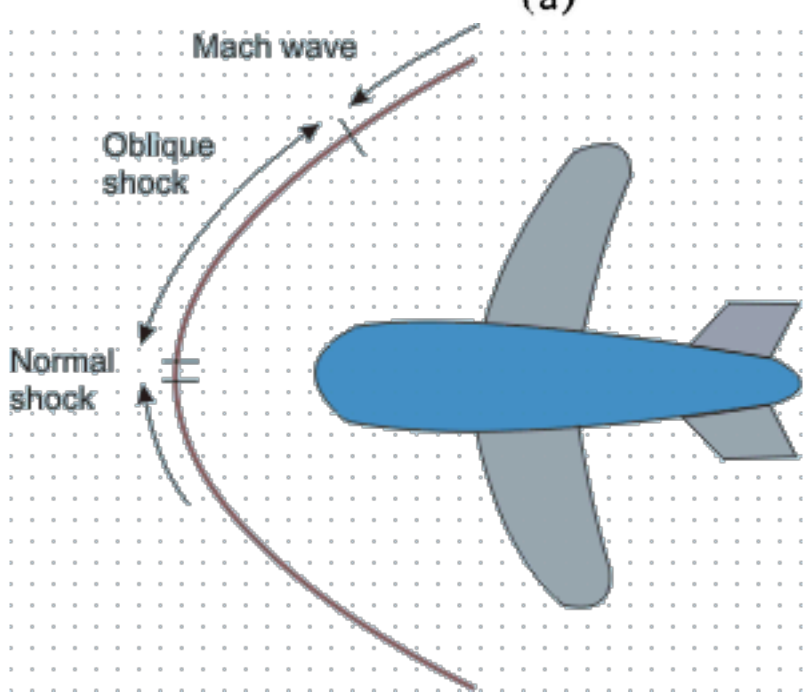
**The deriving process please refer to the handout.**

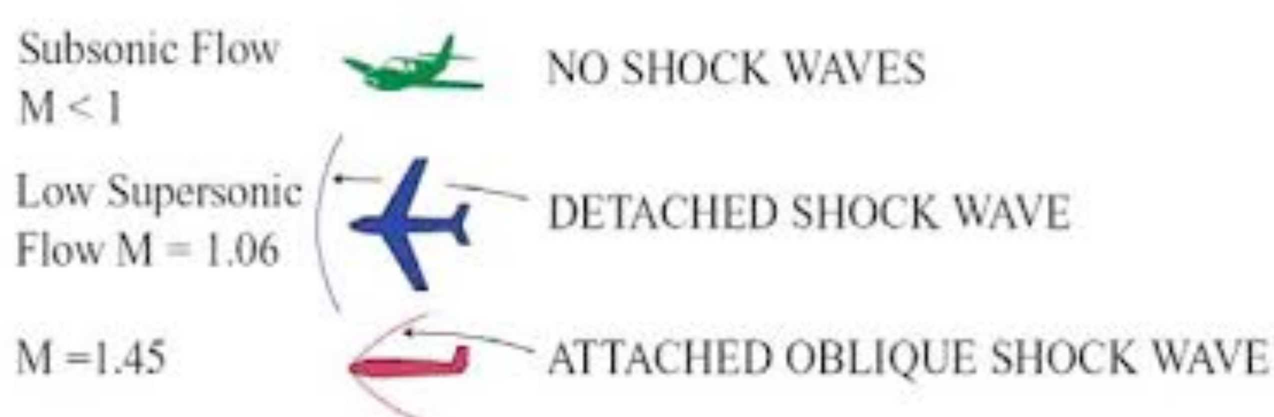
# 6.5 The structure of shock waves




(b)

(a)

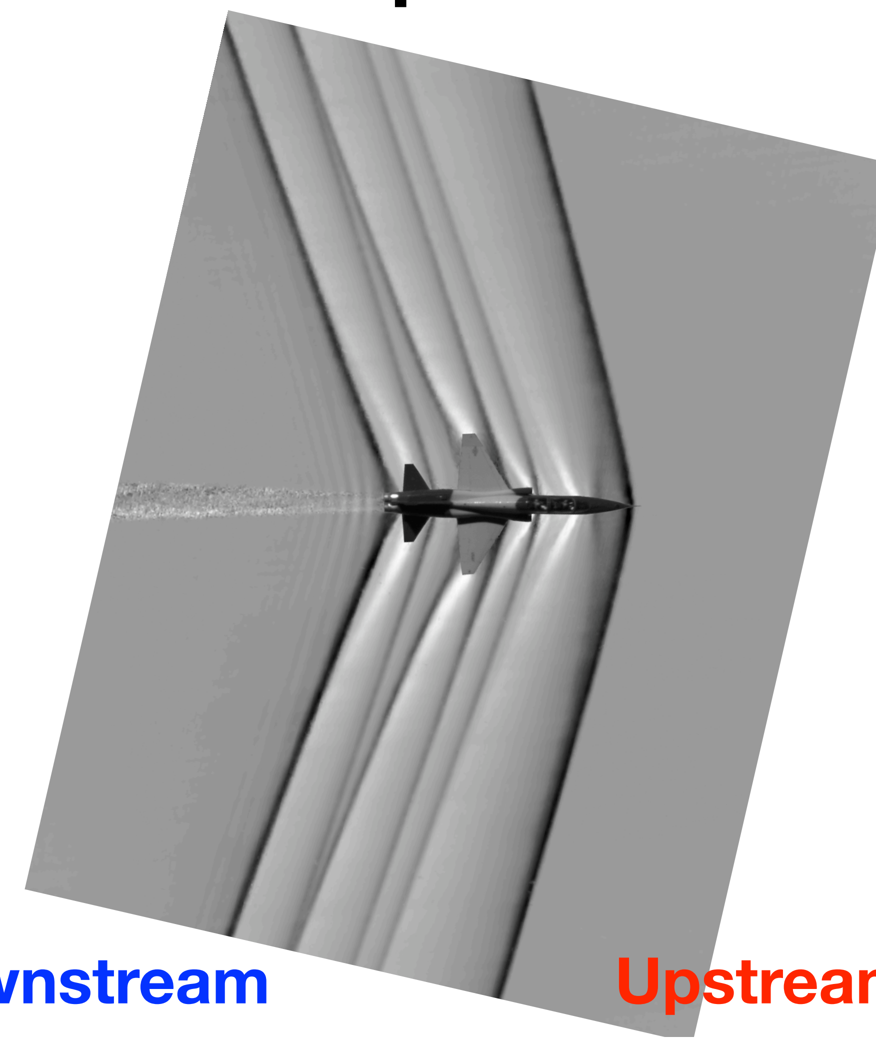


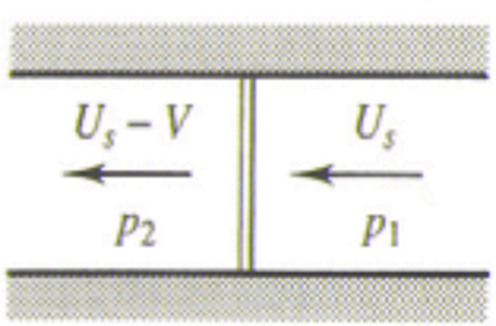
# 6.5 The Moving Normal Shock Waves

**Airplane direction** →

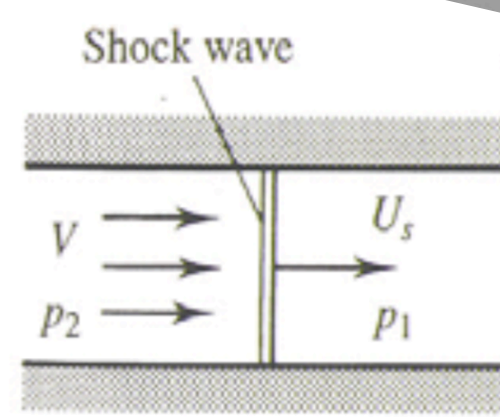
Downstream Upstream



Downstream Upstream




**Shock fixed**




**Shock moving**

# 6.6 Spherical blast waves. Supernova explosions

**Suppose an energy E is suddenly released in an explosion producing a spherical blast wave:**

Scale parameter of the blast wave at time t:

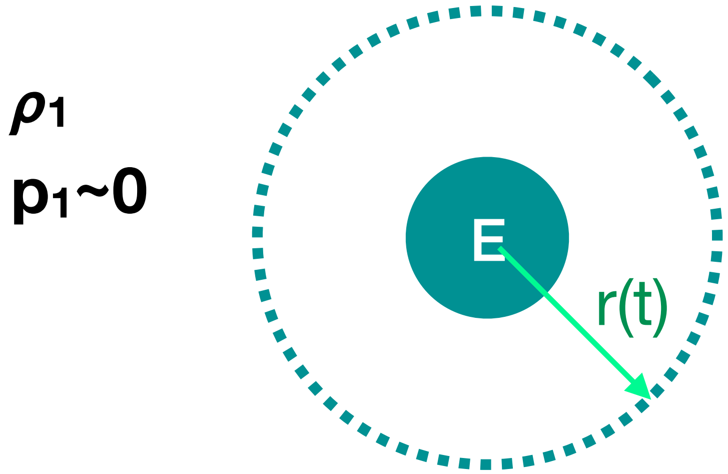
$$\lambda = (Et^2/\rho_1)^{1/5} \qquad (6.40)$$

A dimensionless distance parameter:

$$\xi = \frac{r}{\lambda} = r(\frac{\rho_1}{Et^2})^{1/5} \qquad (6.41)$$

Radius of the spherical blast:

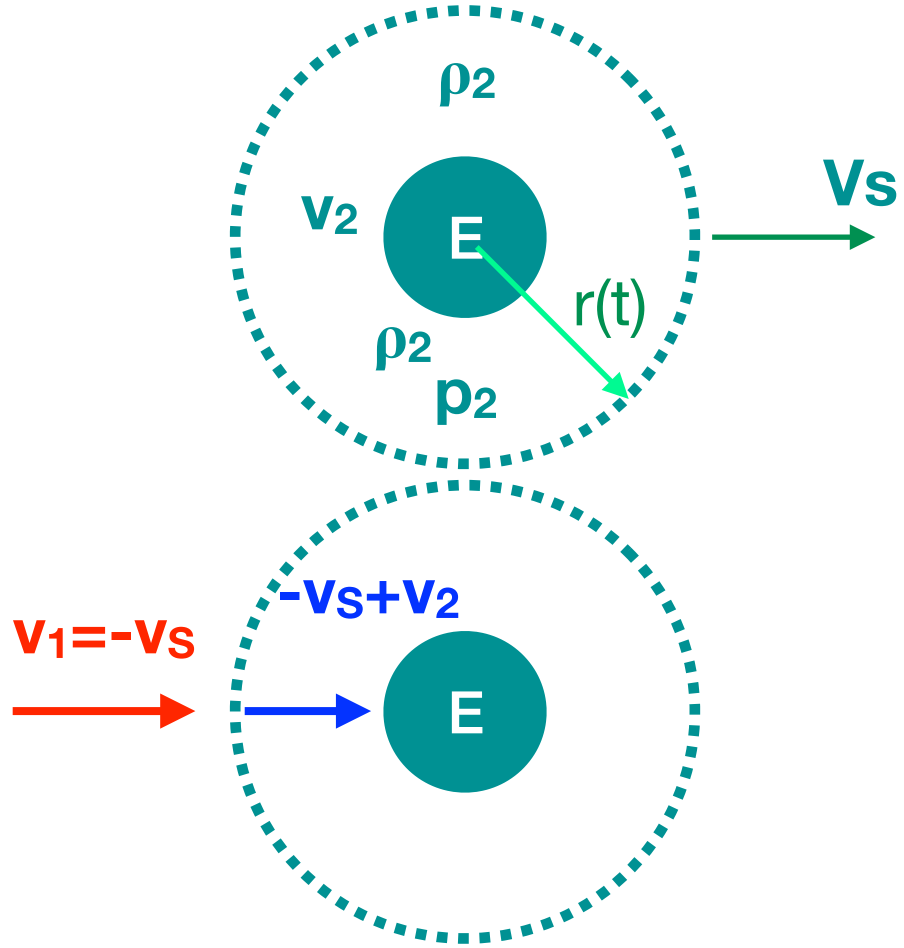
$$r_S(t) = \xi_0(\frac{Et^2}{\rho_1})^{1/5} \qquad (6.42)$$

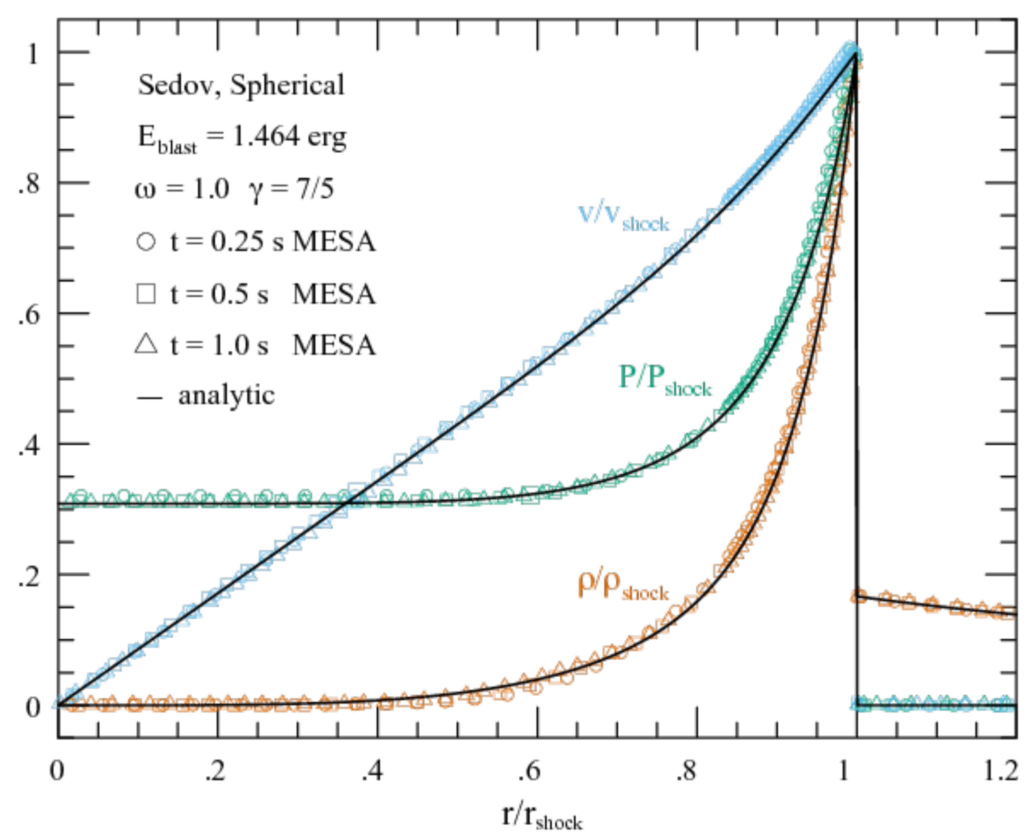



Velocity of expansion:

$$v_S(t) = \frac{dr_S}{dt} = \frac{2r_S}{5t} = \frac{2}{5}\xi_0(\frac{E}{\rho_1 t^3})^{1/5} \qquad (6.43)$$

# 6.6 Spherical blast waves. Supernova explosions




In the frame of the shock:

$$\rho_2 = (\frac{\gamma+1}{\gamma-1})\rho_1 \quad (6.46)$$

$$v_2 = (\frac{2}{\gamma+1})v_S \quad (6.47)$$

$$p_2 = (\frac{2}{\gamma+1})\rho_1 v_S^2 \quad (6.48)$$

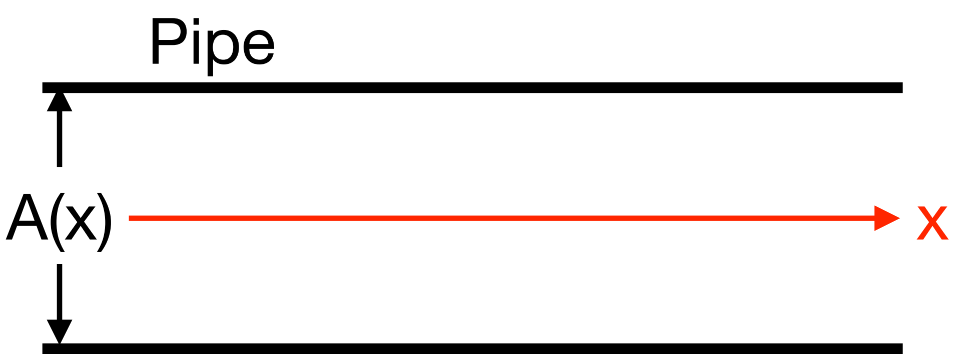
Substitute (6.49-6.51) into (6.53-6.55):

$$\frac{\partial}{\partial t} = -\frac{2\xi}{5t}\frac{d}{d\xi} \qquad \frac{\partial}{\partial r} = \frac{\xi}{r}\frac{d}{d\xi}$$

In terms of dimensionless variables:

$$\frac{32\pi}{25(\gamma^2-1)}\int_0^{\xi_0}[p' + \rho' v'^2]\xi^4 d\xi = 1 \quad (6.59)$$

# 6.7 One-dimensional gas flow. Extragalactic jets




Consider a steady, adiabatic gas flow not vary with time:

$$\frac{d}{dx}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$

$$\frac{dp}{dx} = c_s^2 \frac{d\rho}{dx} \qquad (6.60)$$

Where $$c_s = \sqrt{\frac{\gamma p}{\rho}} \qquad (6.61)$$

Assume the same mass flux pass through the pipe at any x:

$$\rho(x)v(x)A(x) = constant \qquad (6.62)$$

The Euler equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (6.29)$$

The x is the only independent variable, and put the equation (6.60) into (6.29):

$$v\frac{dv}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx} \qquad (6.63)$$

Differentiating (6.62):

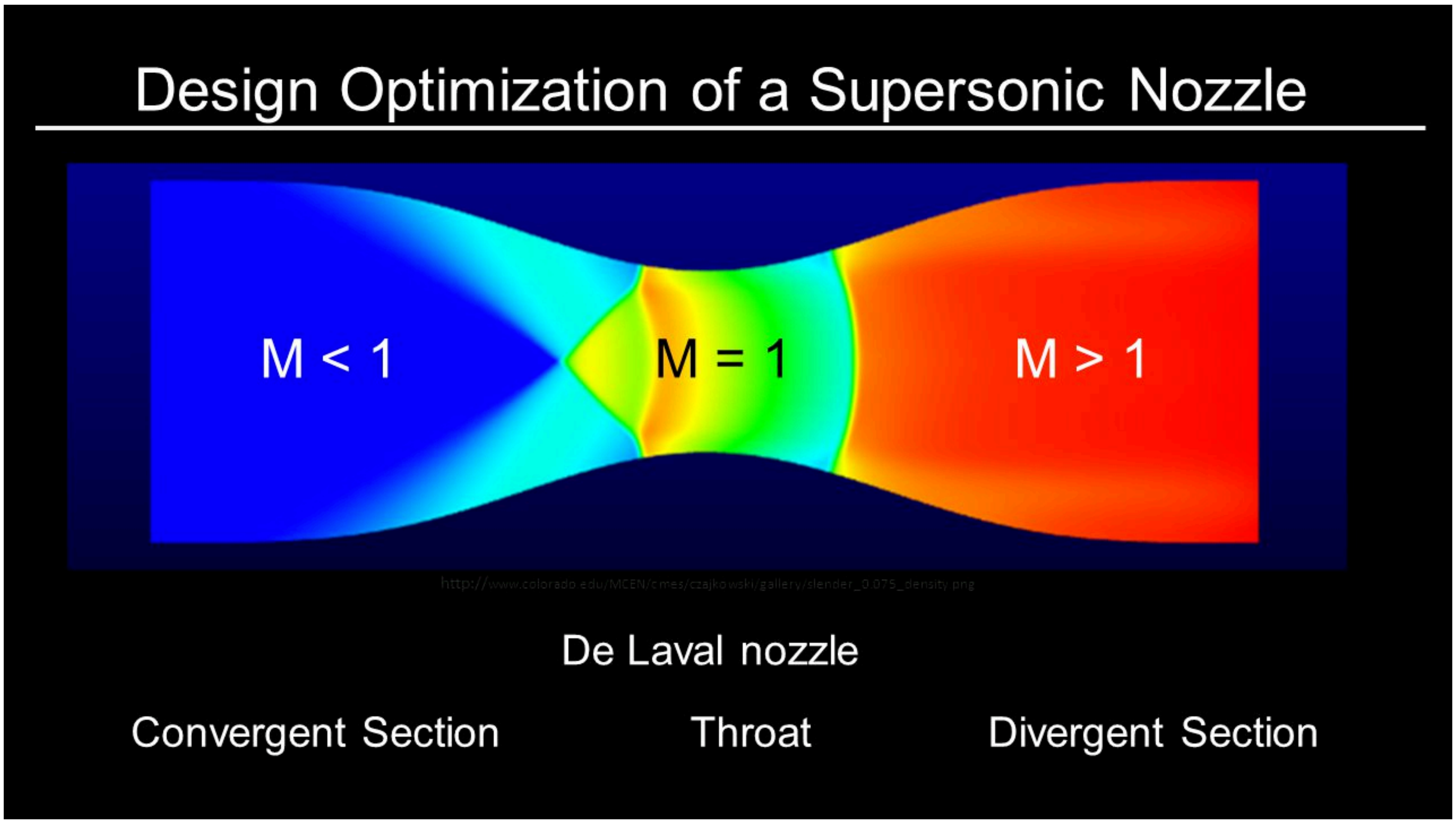
$$\frac{1}{\rho}\frac{\rho}{dx} + \frac{1}{v}\frac{v}{dx} + \frac{1}{A}\frac{A}{dx} = 0 \qquad (6.64)$$

Eliminating (1/$\rho$)(d$\rho$/dx):

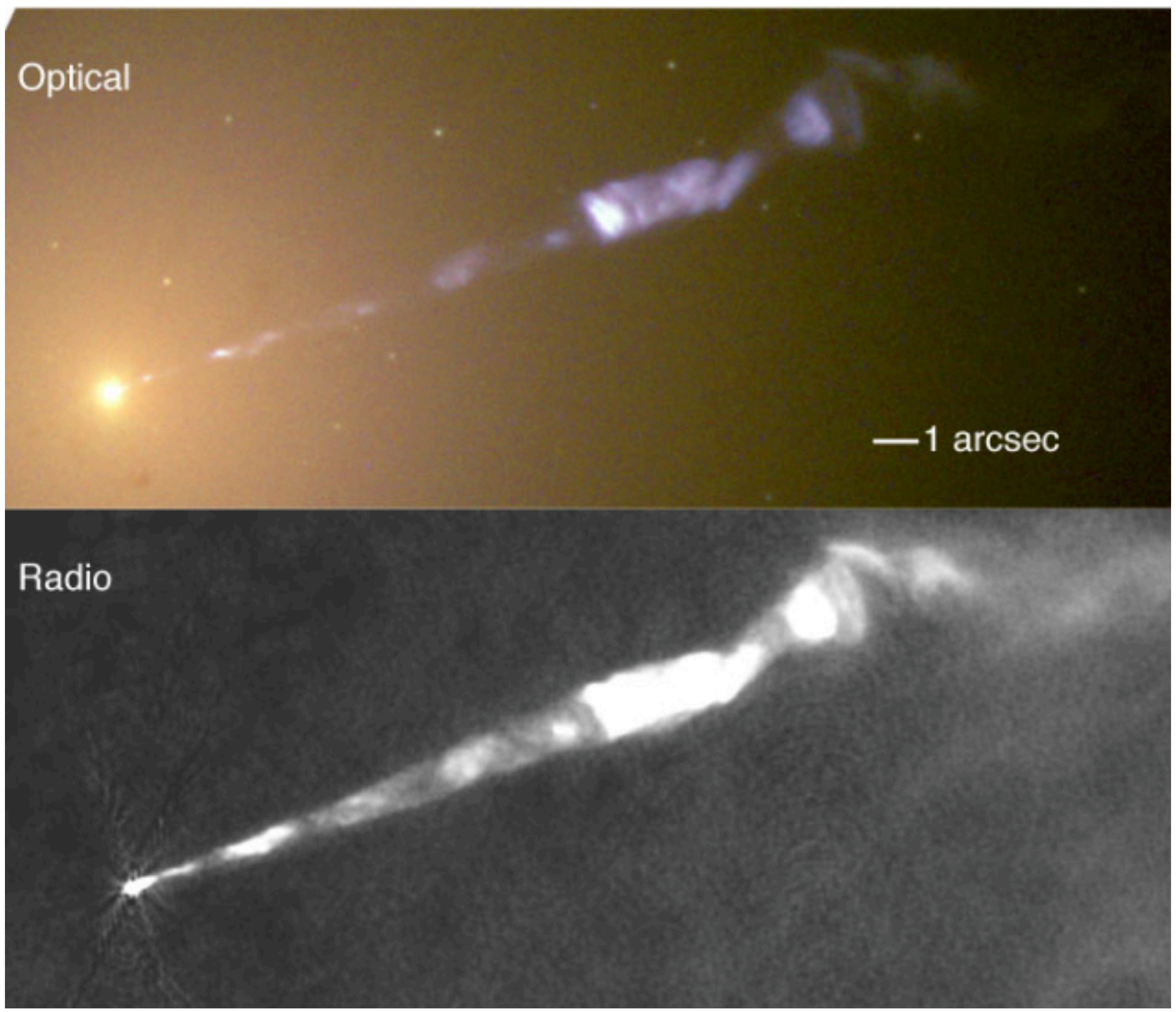
$$(1 - \mathscr{M}^2)\frac{1}{v}\frac{v}{dx} = -\frac{1}{A}\frac{A}{dx} \qquad (6.65)$$

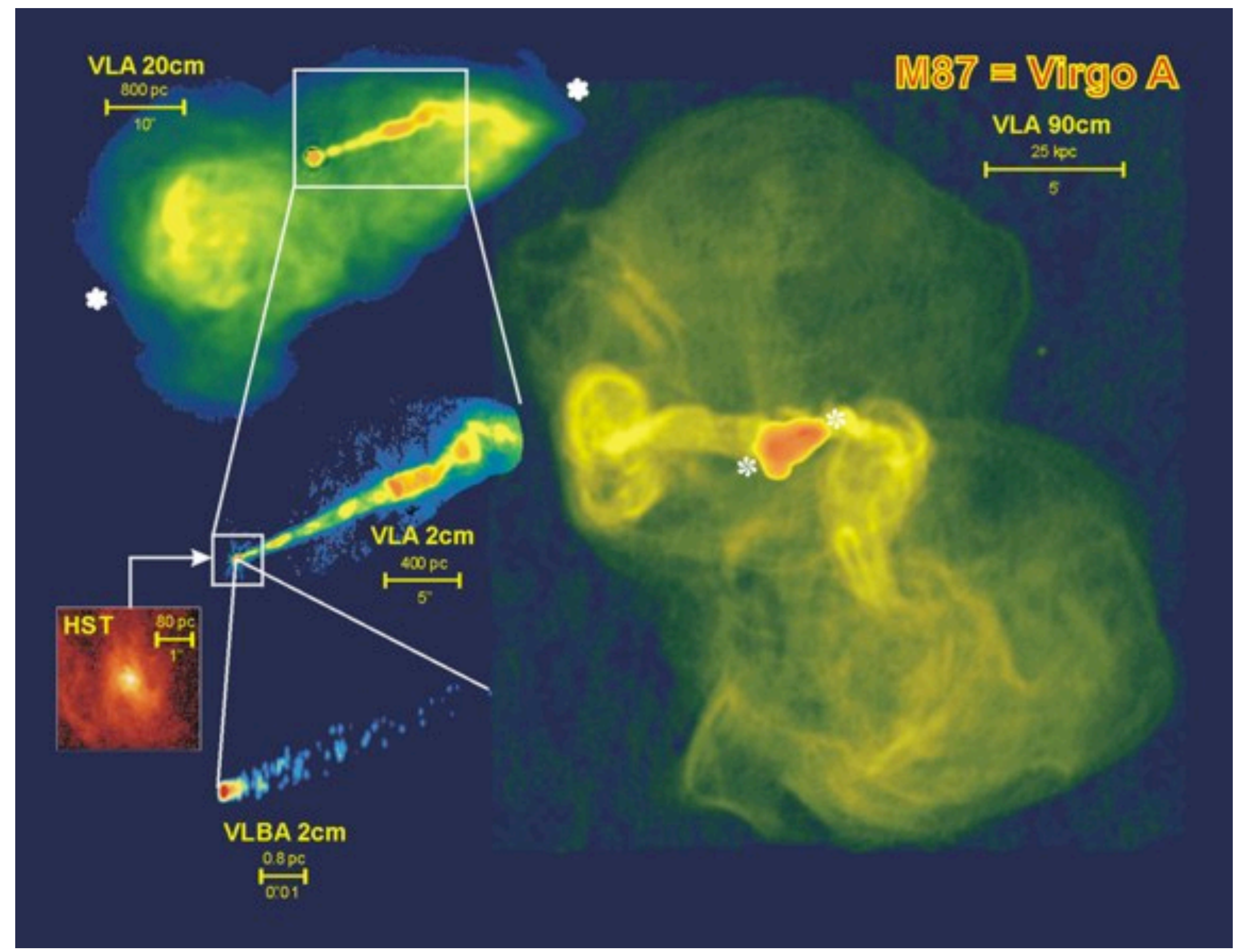
Where $\mathscr{M} = \frac{v_1}{c_{s,1}}$

# 6.7 One-dimensional gas flow. Extragalactic jets

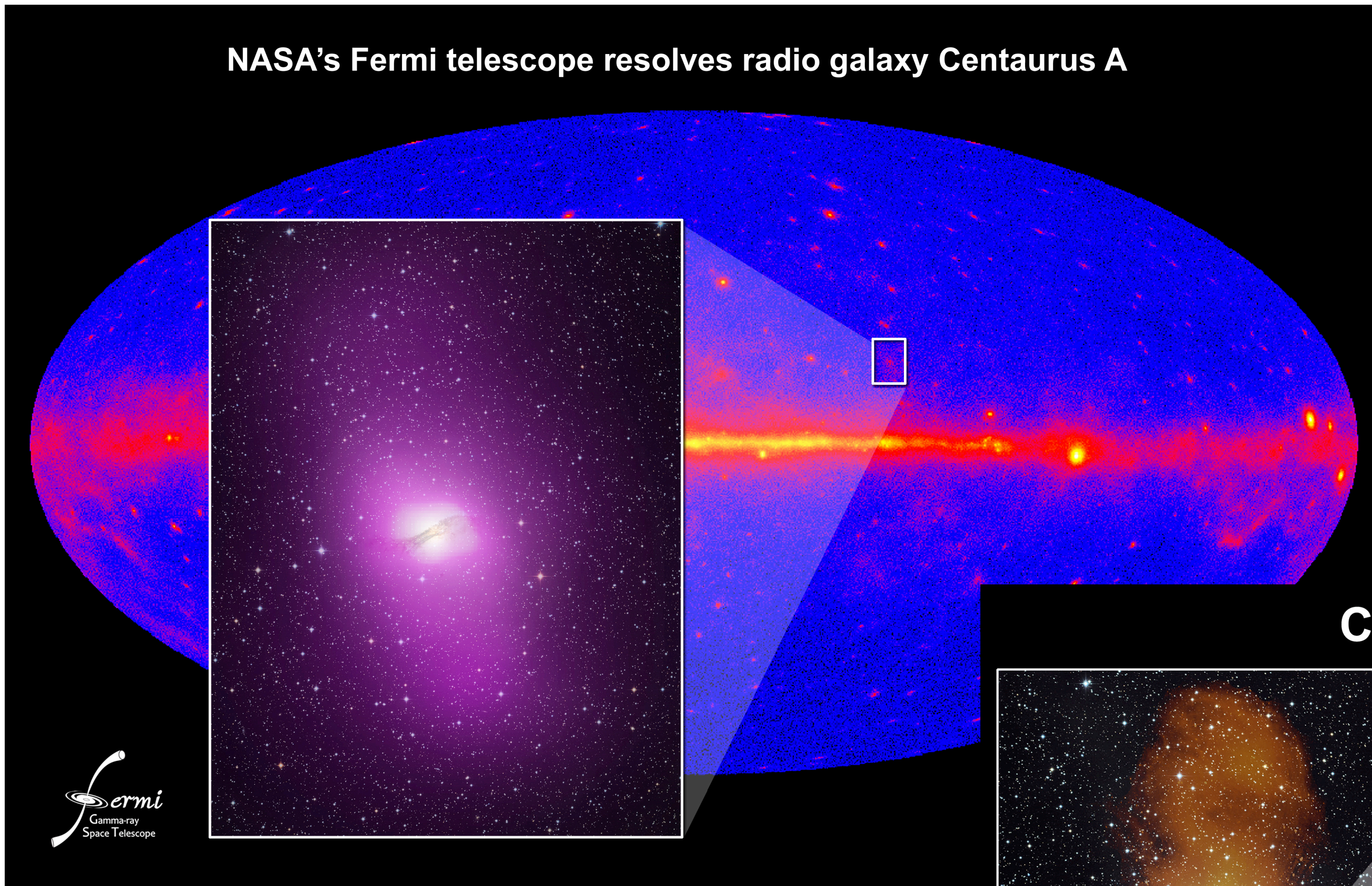


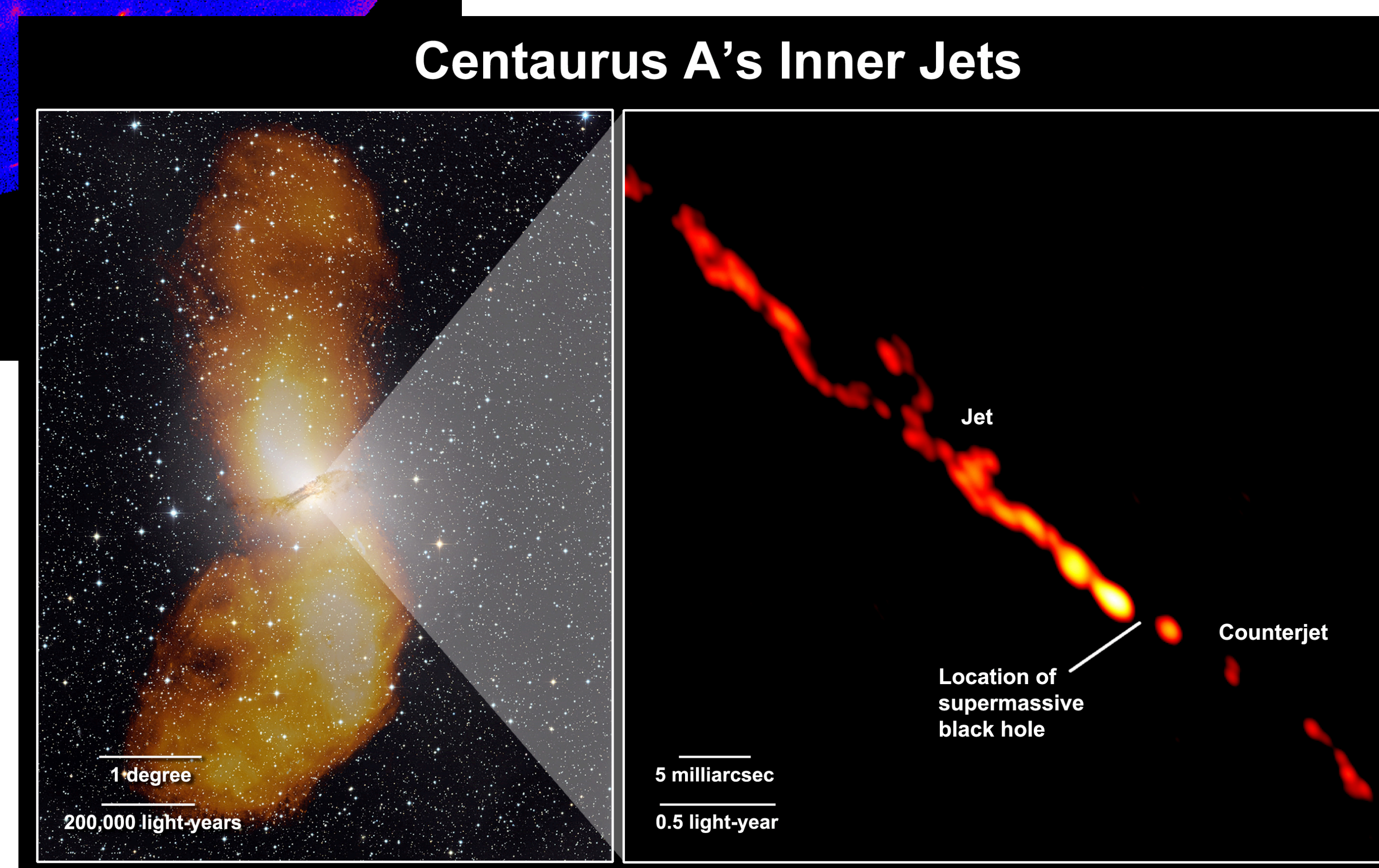
# 6.7 One-dimensional gas flow. Extragalactic jets (M87)



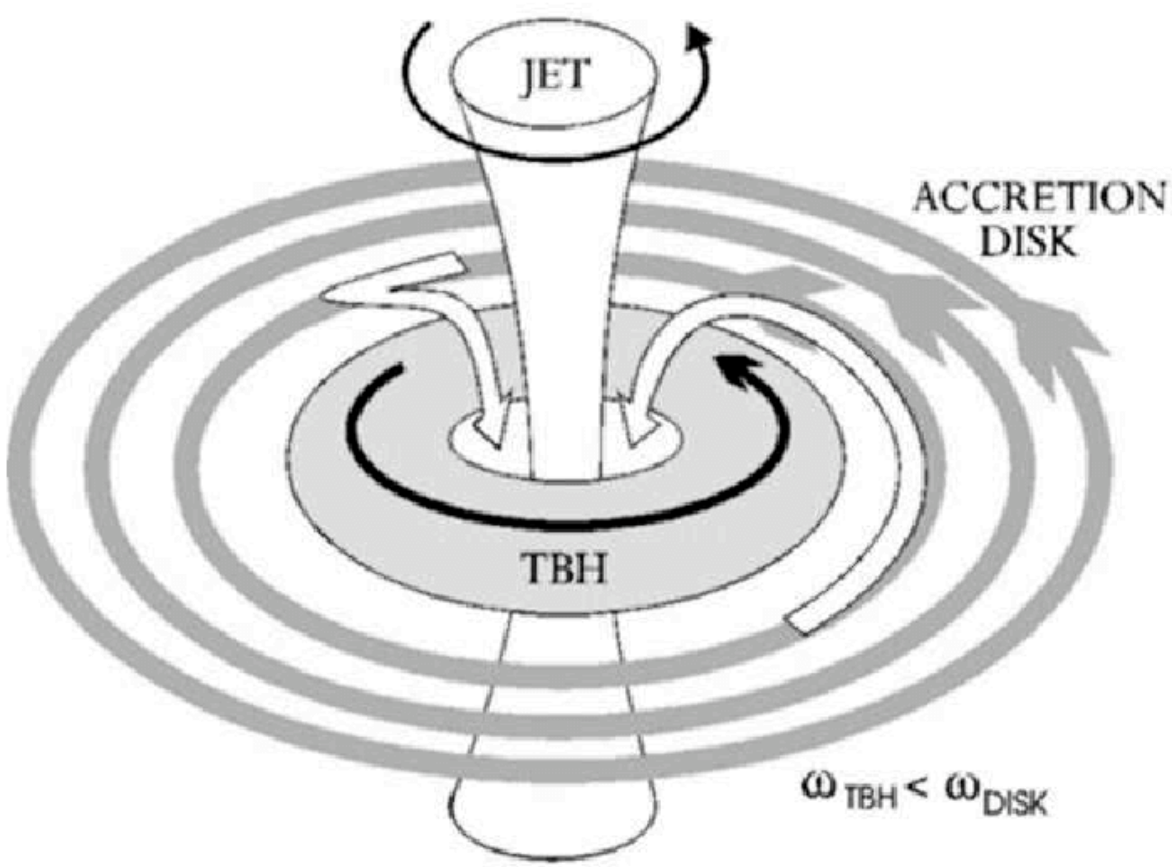



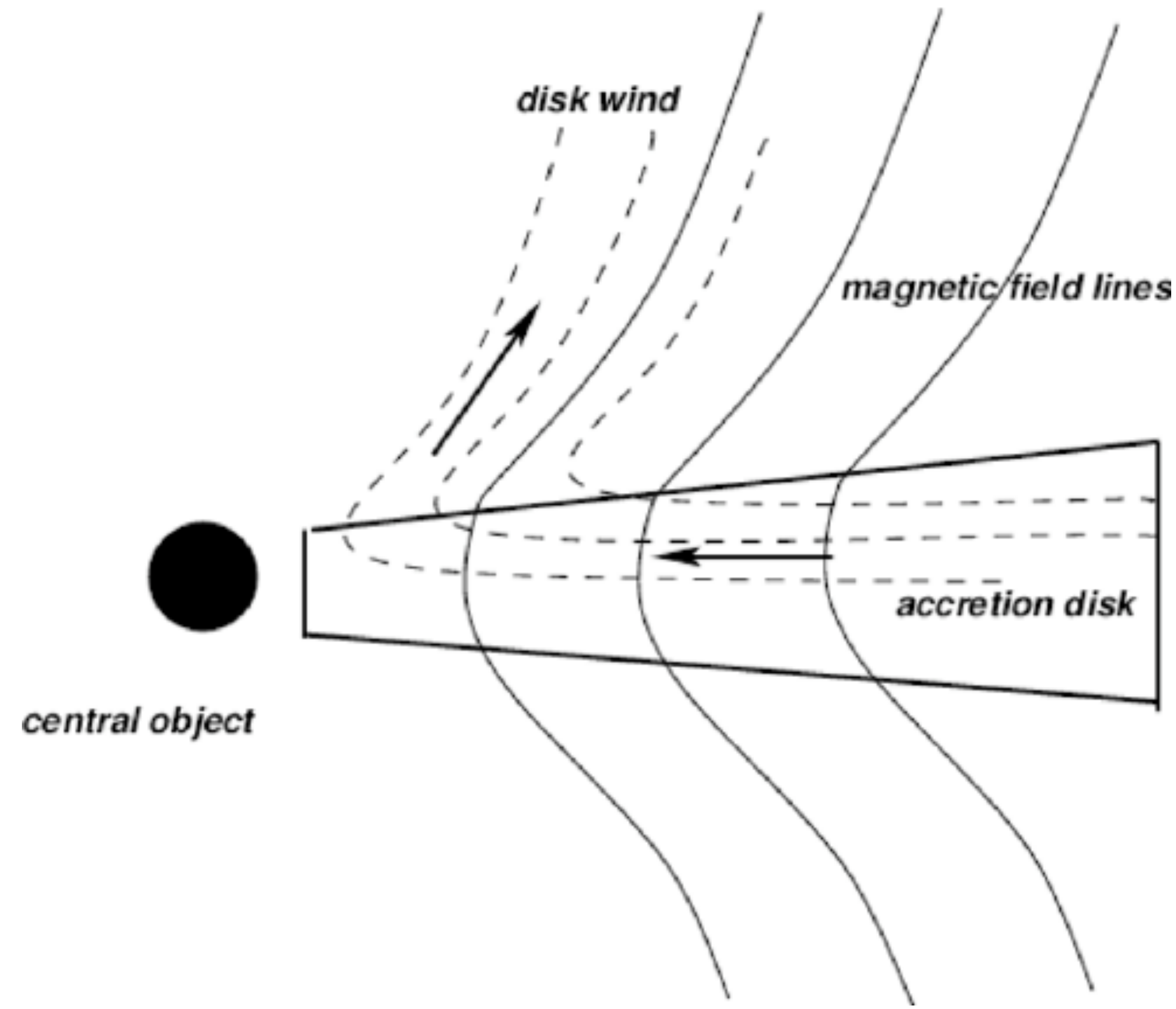
# 6.7 One-dimensional gas flow. Extragalactic jets (Cen A)



# 6.8 Spherical accretion and winds



# 6.8 Spherical accretion and winds