2.4 Boltzman's H theorem (参考資料)

• $\frac{dH}{dt} = \frac{d}{dt}\int d^3u\ f\log f$

$= \cancel{\frac{d}{dt}} \int d^3u\ \frac{\partial}{\partial t}(f\log f)$

$= \int d^3u \cdot \frac{\partial f}{\partial t}\cdot\frac{d}{df}(f\log f) = \int d^3u\ \frac{\partial f}{\partial t}(1+\log f)$

• $\frac{dH}{dt} = \int d^3u_1 \int d^3u \int d\Omega\ \sigma(\Omega)\,|u_1-u|\,(f_1'f'-f_1 f)(1+\log f_1)$

を(2.27)と足すと.

$2\frac{dH}{dt} = \int d^3u\int d^3u_1 \cdots\cdots (f'f_1-ff_1)(2+\log f+\log f_1) \to (2.28)$

($\overset{1}{\to}$ $\overset{2}{\to}$ と $\overset{2}{\to}$ $\overset{1}{\to}$ は同じ?)

• $(\beta-\alpha)\log\left(\frac{\alpha}{\beta}\right) \le 0$ の証明

$\beta>0,\ \alpha>0$ とする.

・$\alpha=\beta$ のとき, 0

・$\alpha>\beta$ 〃 $(-)\log(1より大) < 0$ $\oplus$

・$\alpha<\beta$ 〃 $(+)\log(1より小) < 0$ $\ominus$

∴ $(\beta-\alpha)\log\left(\frac{\alpha}{\beta}\right) \le 0$ □

• $\{\vec u, \vec u_1\} \rightleftarrows \{\vec u', \vec u_1'\}$ より (時間反転)

$2\frac{dH}{dt} = \underbrace{\int d^3u'\int d^3u_1'}_{(2.12)} \int d\Omega\ \sigma(\Omega)\underbrace{|\vec u'-\vec u_1'|}_{(2.11)} \underbrace{(ff_1 - f'f_1')}_{\uparrow\ 符号が(2.28)と逆} [2+\log(f'f_1')] \to (2.29)$

(2.28) + (2.29)

$4\frac{dH}{dt} = \int d^3u\int d^3u_1\int d\Omega\ \sigma(\Omega)\,|\vec u-\vec u_1|\,(f'f_1'-ff_1)[\cancel{2}+\log(ff_1)\cancel{-2}-\log(f'f_1')]$

→ (2.30)

2.5 The conservation eq (参考資料)

• (2.15) は.

$$\frac{\partial f}{\partial t} + \vec{u}\cdot\nabla f + \frac{\vec{F}}{m}\cdot\nabla_u f = \int d^3u_1 \int d\Omega\, |\vec{u}-\vec{u}_1|\, \sigma(\Omega)\, (f'f_1' - f f_1)$$

• $\chi \to \chi_1$ として, (2.32) は.

$$(右) = \int d^3u_1 \int d^3u \int d\Omega\, \sigma(\Omega)\, |\vec{u}_1-\vec{u}|\, (f_1'f' - f_1 f)\, \chi_1$$

このとき,

$$(左) = \underbrace{\left(\frac{\partial f_1}{\partial t} + \vec{u}_1\cdot\nabla f_1 + \frac{\vec{F}}{m}\cdot\nabla_{u_1} f_1\right)}_{\chi_1} \overset{?}{=} \left(\frac{\partial f}{\partial t} + \vec{u}\cdot\nabla f + \frac{\vec{F}}{m}\cdot\nabla_u f.\right)\chi$$

• (2.34) → (2.35)

$$(2.34) \to \underbrace{\int d^3u\, \chi \frac{\partial f}{\partial t}}_{①} + \underbrace{\int d^3u\, \chi u_i \frac{\partial f}{\partial x_i}}_{②} + \underbrace{\int d^3u\, \chi \frac{F_i}{m} \frac{\partial f}{\partial u_i}}_{③} = 0$$

$$① = \frac{\partial}{\partial t}\int d^3u\, \chi f.$$

$$② = \int d^3u \cdot u_i \frac{\partial}{\partial x_i}(\chi f) - \int d^3u\, u_i f \frac{\partial \chi}{\partial x_i}$$

$$= \frac{\partial}{\partial x_i}\int d^3u\, \chi u_i f - \int d^3u\, u_i f \frac{\partial \chi}{\partial x_i}$$

$$③ = \frac{1}{m}\int d^3u \frac{\partial}{\partial u_i}(\chi F_i f) - \frac{1}{m}\int d^3u \frac{\partial \chi}{\partial u_i} F_i f - \frac{1}{m}\int d^3u\, \chi \frac{\partial F_i}{\partial u_i} f$$

m